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STRONG INNER INVERSES IN ENDOMORPHISM RINGS OF VECTOR SPACES

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Abstract: For V a vector space over a field, or more generally, over a division ring, it is well-known that every $x \in \mathrm{End}(V)$ has an *inner inverse*; that is, that there exists $y \in \mathrm{End}(V)$ satisfying $xyx = x$. We show here that a large class of such x have inner inverses y that satisfy with x an infinite family of additional monoid relations, making the monoid generated by x and y what is known as an *inverse monoid* (definition recalled). We obtain consequences of these relations, and related results.

P. Nielsen and J. Šter [**16**] show that a much larger class of elements x of rings R, including all elements of von Neumann regular rings, have inner inverses satisfying arbitrarily large *finite* subsets of the abovementioned set of relations. But we show by example that the endomorphism ring of any infinite-dimensional vector space contains elements having no inner inverse that simultaneously satisfies all those relations.

A tangential result gives a condition on an endomap x of a set S that is necessary and sufficient for x to have a strong inner inverse in the monoid of all endomaps of S.

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