Publ. Mat. **65** (2021), 3–59
DOI: 10.5565/PUBLMAT6512101

UNIQUENESS PROPERTY FOR 2-DIMENSIONAL MINIMAL CONES IN $\mathbb{R}^3$

Xiangyu Liang

Abstract: In this article we treat two closely related problems: 1) the upper semi-continuity property for Almgren minimal sets in regions with regular boundary; and 2) the uniqueness property for all the 2-dimensional minimal cones in $\mathbb{R}^3$.

Given an open set $\Omega \subset \mathbb{R}^n$, a closed set $E \subset \Omega$ is said to be Almgren minimal of dimension d in Ω if it minimizes the d-Hausdorff measure among all its Lipschitz deformations in Ω. We say that a d-dimensional minimal set E in an open set Ω admits upper semi-continuity if, whenever $\{f_n(E)\}_n$ is a sequence of deformations of E in Ω that converges to a set F, then we have $\mathcal{H}^d(F) \geq \limsup_n \mathcal{H}^d(f_n(E))$. This guarantees in particular that E minimizes the d-Hausdorff measure, not only among all its deformations, but also among limits of its deformations.

As proved in [**19**], when several 2-dimensional minimal cones are all translational and sliding stable, and admit the uniqueness property, then their almost orthogonal union stays minimal. As a consequence, the uniqueness property obtained in the present paper, together with the translational and sliding stability properties proved in [**18**] and [**20**] permit us to use all known 2-dimensional minimal cones in $\mathbb{R}^n$ to generate new families of minimal cones by taking their almost orthogonal unions.

The upper semi-continuity property is also helpful in various circumstances: when we have to carry on arguments using Hausdorff limits and some properties do not pass to the limit, the upper semi-continuity can serve as a link. As an example, it plays a very important role throughout [**19**].

2010 Mathematics Subject Classification: 28A75, 49Q20, 49K21.

Key words: minimal cones, uniqueness, Hausdorff measure, Plateau's problem.