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CLASSICAL AND UNIFORM EXPONENTS OF MULTIPLICATIVE p-ADIC APPROXIMATION

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Abstract: Let p be a prime number and ξ an irrational p-adic number. Its irrationality exponent $\mu(\xi)$ is the supremum of the real numbers μ for which the system of inequalities

$$0 < \max\{|x|, |y|\} \leq X, \quad |y\xi - x|_p \leq X^{-\mu}$$

has a solution in integers x, y for arbitrarily large real number X. Its multiplicative irrationality exponent $\mu^{\times}(\xi)$ (resp., uniform multiplicative irrationality exponent $\widehat{\mu}^{\times}(\xi)$) is the supremum of the real numbers $\widehat{\mu}$ for which the system of inequalities

$$0 < |xy|^{1/2} \leq X, \quad |y\xi - x|_p \leq X^{-\widehat{\mu}}$$

has a solution in integers x, y for arbitrarily large (resp., for every sufficiently large) real number X. It is not difficult to show that $\mu(\xi) \leq \mu^{\times}(\xi) \leq 2\mu(\xi)$ and $\widehat{\mu}^{\times}(\xi) \leq 4$. We establish that the ratio between the multiplicative irrationality exponent $\mu^{\times}$ and the irrationality exponent μ can take any given value in $[1, 2]$. Furthermore, we prove that $\widehat{\mu}^{\times}(\xi) \leq (5+\sqrt{5})/2$ for every p-adic number ξ.

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Key words: rational approximation, p-adic number, exponent of approximation.