NON-EXISTENCE OF INTEGRAL HOPF ORDERS FOR TWISTS OF SEVERAL SIMPLE GROUPS OF LIE TYPE

GIOVANNA CARNOVALE, JUAN CUADRA, AND ELISABETTA MASUT

Abstract: Let p be a prime number and $q = p^m$, with $m \ge 1$ if $p \ne 2,3$ and m > 1 otherwise. Let Ω be any non-trivial twist for the complex group algebra of $\mathbf{PSL}_2(q)$ arising from a 2-cocycle on an abelian subgroup of $\mathbf{PSL}_2(q)$. We show that the twisted Hopf algebra $(\mathbb{C}\mathbf{PSL}_2(q))_{\Omega}$ does not admit a Hopf order over any number ring. The same conclusion is proved for the Suzuki groups, and for $\mathbf{SL}_3(p)$ when the twist stems from an abelian p-subgroup. This supplies new families of complex semisimple (and simple) Hopf algebras that do not admit a Hopf order over any number ring. The strategy of the proof is formulated in a general framework that includes the finite simple groups of Lie type.

As an application, we combine our results with two theorems of Thompson and Barry and Ward on minimal simple groups to establish that for any finite non-abelian simple group G there is a twist Ω for $\mathbb{C}G$, arising from a 2-cocycle on an abelian subgroup of G, such that $(\mathbb{C}G)_{\Omega}$ does not admit a Hopf order over any number ring. This partially answers in the negative a question posed by Meir and the second author.

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Key words: semisimple Hopf algebra, Drinfeld twist, Hopf order, finite group of Lie type.