TEST VECTORS FOR ARCHIMEDEAN PERIOD INTEGRALS

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Abstract: We study period integrals involving Whittaker functions associated to generic irreducible Casselman–Wallach representations of $\mathrm{GL}_n(F)$, where F is an archimedean local field. Via the archimedean theory of newforms for GL_n developed by the first author, we prove that newforms are weak test vectors for several period integrals, including the $\mathrm{GL}_n \times \mathrm{GL}_n$ Rankin–Selberg integral, the Flicker integral, and the Bump–Friedberg integral. By taking special values of these period integrals, we deduce that newforms are weak test vectors for Rankin–Selberg periods, Flicker–Rallis periods, and Friedberg–Jacquet periods. These results parallel analogous results in the nonarchimedean setting proved by the second author, which use the nonarchimedean theory of newforms for GL_n developed by Jacquet, Piatetski-Shapiro, and Shalika. By combining these archimedean and nonarchimedean results, we prove the existence of weak test vectors for certain global period integrals of automorphic forms.