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CORRIGENDA: "STABILIZATION IN $H^\infty_{\mathbb{R}}(\mathbb{D})$"

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Abstract

In this corrigenda we outline the necessary changes to the paper [**3**] so that the main result in that paper is made correct. The mistake the author made in the previous version was that the condition that f_1 being positive on the zeros of f_2 was not strong enough to guarantee the existence of the logarithm in $H^\infty_{\mathbb{R}}(\mathbb{D})$.

In particular, the main result now is the following theorem: Suppose that $f_1, f_2 \in H^\infty_{\mathbb{R}}(\mathbb{D})$, with $\|f_1\|_\infty, \|f_2\|_\infty \leq 1$, with

$$\inf_{z\in\mathbb{D}} (|f_1(z)| + |f_2(z)|) = \delta > 0.$$

Assume for some $\epsilon > 0$, f_1 has the same sign on the set $\{x \in (-1,1) : |f_2(x)| < \epsilon\}$. Then there exists $g_1, g_1^{-1}, g_2 \in H^\infty_{\mathbb{R}}(\mathbb{D})$ with $\|g_1\|_\infty, \|g_2\|_\infty, \|g_1^{-1}\|_\infty \leq C(\delta,\epsilon)$ and

$$f_1(z)g_1(z) + f_2(z)g_2(z) = 1 \quad \forall\, z \in \mathbb{D}.$$

1. Introduction

In the original paper [**3**] the proof of the main result is unfortunately incorrect, however the method of proof is the correct approach. In this corrigenda, we outline the modifications necessary to modify the existing proof to be correct. The full version of the paper with the necessary corrections has been posted to the `http://arxiv.org/abs/0809.1573`.

The problem with the main result in [**3**] is that the initial hypothesis was that f_1 was of the same sign on the real zeros of f_2. This condition, while necessary is unfortunately not strong enough to be sufficient and to guarantee that the logarithm will exist in $H^\infty_{\mathbb{R}}(\mathbb{D})$. First, we need to extend the definition of positive on zeros. This is accomplished by the following lemma.

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Lemma 1.1. *Let* $f_1, f_2 \in H^\infty_{\mathbb{R}}(\mathbb{D})$ *and suppose that there exist* $g_1, g_1^{-1}, g_2 \in H^\infty_{\mathbb{R}}(\mathbb{D})$ *such that*

$$f_1 g_1 + f_2 g_2 = 1 \quad \forall\, z \in \mathbb{D}.$$

Then, for some $\epsilon > 0$ *we have that* f_1 *has the same sign on the set* $\{x \in (-1,1) : |f_2(x)| < \epsilon\}$.

Proof: The proof of this lemma is a quantification and repetition of the argument given in [**3**] and one can see that argument shows that f_1 has the same sign on the set $\{x \in (-1,1) : |f_2(x)| < \epsilon\}$. We provide the details now. Choose any $0 < \epsilon < \|g_2\|_\infty^{-1}$. With this choice we have that

$$1 - f_2(x) g_2(x) > 0 \quad \text{on } \{x \in (-1,1) : |f_2(x)| < \epsilon\}.$$

If f_1 is not of the same sign on the set $\{x \in (-1,1) : |f_2(x)| < \epsilon\}$ then there exists two points x_1 and x_2 in $\{x \in (-1,1) : |f_2(x)| < \epsilon\}$ such that $f_1(x_1) > 0 > f_1(x_2)$.

We then have

$$1 - f_2(x_1) g_2(x_1) = f_1(x_1) g_1(x_1)$$
$$1 - f_2(x_2) g_2(x_2) = f_1(x_2) g_1(x_2)$$

and so $g_1(x_1) > 0 > g_1(x_2)$ since the same inequality holds for f_1 at the points x_1 and x_2. But, this is clearly a contradiction to the hypothesis that $g_1^{-1} \in H^\infty_{\mathbb{R}}(\mathbb{D})$ since by continuity there must exist a point x_{12} between x_1 and x_2 such that $g_1(x_{12}) = 0$ which is forbidden since g_1 is invertible in $H^\infty_{\mathbb{R}}(\mathbb{D})$. □

With this condition, we now state the correct form of the main theorem from [**3**].

Theorem 1.2. *Suppose that* $f_1, f_2 \in H^\infty_{\mathbb{R}}(\mathbb{D})$, $\|f_1\|_\infty, \|f_2\|_\infty \leq 1$. *Assume that for some* $\epsilon > 0$, f_1 *has the same sign on the set* $\{x \in (-1,1) : |f_2(x)| < \epsilon\}$ *and*

$$\inf_{z \in \mathbb{D}} (|f_1(z)| + |f_2(z)|) = \delta > 0.$$

Then there exists $g_1, g_1^{-1}, g_2 \in H^\infty_{\mathbb{R}}(\mathbb{D})$ *with* $\|g_1\|_\infty, \|g_2\|_\infty, \|g_1^{-1}\|_\infty \leq C(\delta, \epsilon)$ *and*

$$f_1(z) g_1(z) + f_2(z) g_2(z) = 1 \quad \forall\, z \in \mathbb{D}.$$

Here the constant $C(\delta, \epsilon)$ depends on the condition arising from f_1 being positive on the set $\{x \in (-1,1) : |f_2(x)| < \epsilon\}$. The original statement of the theorem did not take this parameter ϵ into consideration.

Acknowledgements. The author thanks Raymond Mortini for providing interesting counterexamples to the original version of theorem which showed that, quite surprisingly, the constants $C(\delta)$ bounding the norms of the solutions (g_1, g_2) and the inverse g_1^{-1} (in $H^\infty_{\mathbb{R}}(\mathbb{D})$) to the Bezout equation $g_1 f_1 + g_2 f_2 = 1$ not only depend on δ as claimed, but also must depend on a stronger condition about the positivity, namely the parameter ϵ. The author also thanks Kalle Mikkola for a similar observation. Finally, the author thanks Sergei Treil for a helpful discussion.

2. Outline of the corrections

We first remark that the condition that f_1 being positive on the set $\{x \in (-1, 1) : |f_2(x)| < \epsilon\}$ is clearly necessary by Lemma 1.1. First note that if f_1 has the same sign on the set $\{x \in (-1, 1) : |f_2(x)| < \epsilon\}$, then by multiplying the function f_1 by -1 if necessary we can assume that f_1 is positive there. Next observe that if f_1 is positive on the set $\{x \in (-1, 1) : |f_2(x)| < \epsilon\}$ then we will have that f_1 is positive on the real zeros of f_2. We now transfer the construction and conditions to the upper half plane $\mathbb{C}_+$ where certain computations are easier. Note that in this case the condition is that f_1 is positive on the set $\{y \in \mathbb{R}_+ : |f_2(iy)| < \epsilon\}$.

It remains to address the necessary corrections to the delicate construction to the proof appearing in [**3**]. The proof of Theorem 1.2 can be reduced to proving the corresponding result for finite Blaschke products that are real symmetric and possess the extended positivity on the real zeros.

Theorem 2.1. *Let B be a real symmetric Blaschke product with simple zeros, and let σ denote its zero set. Given $0 < \gamma < 1$, let φ be a real symmetric analytic function on the set $\{z : |B(z)| < \gamma\}$ and satisfying $|\varphi(z)| \leq 1$ there. Then there exists a real symmetric function $h \in H^\infty_{\mathbb{R}}(\mathbb{C}_+)$ such that*

$$\varphi(z) = h(z) \quad \forall\, z \in \sigma.$$

Moreover, $\|h\|_\infty \leq C(\gamma)\|\varphi\|_\infty$.

We remark that it suffices to have φ symmetric only on the zeros of the function B, however, we state the result in a slightly stronger form. To prove Theorem 2.1, one simply takes the resulting function that exists in $H^\infty(\mathbb{C}_+)$ (as demonstrated by Carleson in [**1**]), call it $l(z)$, and then symmetrize it by setting $h(z) = \frac{l(z)+\overline{l(-\overline{z})}}{2}$. Since l does the interpolation and everything is symmetric, the result then follows.

Since f_1 is rational, we have a bounded branch of the logarithm $\log f_1$ on the set $\{z : |f_2(z)| < \delta'\}$. Additionally, by choice of $\epsilon > 0$ we have that f_1 is positive on the the set $\{y \in \mathbb{R}_+ : |f_2(iy)| < \epsilon\}$ so we can interpolate the logarithm of f_1 with a function $h \in H^\infty_{\mathbb{R}}(\mathbb{C}_+)$ with $\|h\|_\infty \leq C(\delta, \epsilon)\|\log f_1\|_\infty$, and

$$e^{h(z)} = f_1(z) \quad \text{for all } z \text{ in the zero set of } f_2.$$

The function e^h is invertible in $H^\infty_{\mathbb{R}}(\mathbb{C}_+)$ and there is a function $G \in H^\infty_{\mathbb{R}}(\mathbb{C}_+)$ with $e^h = f_1 + f_2 G$.

This is almost enough to conclude the proof of the theorem. If $\log f_1$ were bounded on $\{z \in \mathbb{C}_+ : |f_2(z)| < \delta'\}$ by a constant only depending on δ and ϵ and *not* on the degrees of f_1 and f_2, we would be done. However, this is not generally true, so we need a method to overcome this difficulty. To do this we will find an analytic function κ that is real symmetric and will "correct" the function f_1. To correct the function f_1 it suffices to prove the following proposition, which has been corrected from [**3**].

Proposition 2.2. *Let $p, q \in H^\infty_{\mathbb{R}}(\mathbb{C}_+)$ be finite simple real symmetric Blaschke products with $\inf_{z\in\mathbb{C}_+}(|p(z)| + |q(z)|) = \delta > 0$ such that for some $\epsilon > 0$, p has the same sign on the set $\{y \in \mathbb{R}_+ : |q(iy)| < \epsilon\}$. Then there exists a function V with the following properties:*

(i) $|\operatorname{Re} V(z)| \leq C(\delta, \epsilon) \quad \forall\, z \in \mathbb{C}_+$;

(ii) $|\log p(z) - V(z)| \leq C(\delta, \epsilon)$ *for all z in $\{z \in \mathbb{C}_+ : |q(z)| < \delta'\}$ for some $0 < \delta'$ sufficiently small with respect to δ and ϵ and an appropriate branch of* $\log p$ *on the set $\{z \in \mathbb{C}_+ : |q(z)| < \delta'\}$;*

(iii) $V(z) = \overline{V(-\overline{z})} \quad \forall\, z \in \mathbb{C}_+$;

(iv) *some conditions to guarantee the existence of a bounded solution v on the entire upper half-plane $\mathbb{C}_+$ of the equation $\bar\partial v = \bar\partial V$, in particular:*

  (a) $|\Delta V(z)| \operatorname{Im} z\, dx\, dy$ *is a Carleson measure with intensity* $C(\delta, \epsilon)$;

  (b) $|\overline{\partial} V(z)|\, dx\, dy$ *is a Carleson measure with intensity* $C(\delta, \epsilon)$;

  (c) $|\Delta V(z)| \leq \frac{C(\delta,\epsilon)}{(\operatorname{Im} z)^2} \quad \forall\, z \in \mathbb{C}_+$.

Once one has constructed this function V then the solution to the Corona problem follows from the same argument as in the original paper. The basic idea in proving Proposition 2.2 was correct and now we must describe how to incorporate the condition that p has the same sign on the set $\{y \in \mathbb{R}_+ : |q(iy)| < \epsilon\}$.

We remark that it is always possible to construct a symmetric function V from the problem. This simply follows from the symmetrization of the proof in [**2**] and that the data p and q have this symmetry property. However, we remark that the positivity condition on p and q is necessary to construct the symmetric branch of logarithm this correcting function V will approximate. It was here that the author made a mistake in the previous version of [**3**].

First, as in the paper [**3**] we construct a symmetric Carleson contour about the zero set of p. Here we will use the same notation as appeared in [**3**]. This resulted in a collection of zeros σ_1 and regions $\mathcal{R}$.

We have to now split the construction of the function V into the cases when the zeros of p are off the imaginary axis and when the zeros are on the imaginary axis. This is necessary so that we can define the appropriate branch of the logarithm. When the zeros are off the axis, the construction is straightforward and one can just take the construction in [**2**] and symmetrize it appropriately, which was what originally appeared in [**3**]. However, when the zeros are on the imaginary axis, we need a different construction, in particular different than what appears in [**3**]. This is a place where the original paper did not have the appropriate construction.

Here is a rough idea about the corrections that need to be made for the construction of the function V. If the region in question had an even number of real zeros, then we would be fine. In this case we have that above and below the region, a certain Blaschke product will be positive, and so we can select a branch of the logarithm so that it is real symmetric. In this case we can appeal to a similar construction to what appears when the regions avoid $i\mathbb{R}_+$.

The difficulty in the construction when the zeros are on the imaginary axis is that they could occur in "groups" with an odd multiplicity. If this happens, then it will be impossible to define the branch of the logarithm that is real symmetric, since above the region we are interested in a related Blaschke factor that will be positive while below the region it will be negative, and so we can not define a branch of the logarithm that is real symmetric. This difficulty will *not* allow us to define the approximating function V in a simple manner, and instead we will have to "pair" certain zeros to overcome this difficulty. The idea will be to pair regions with an odd number of real zeros to make regions with an even number of real zeros so that we can then appeal to the construction when there are an even number of real zeros. Because of hypotheses of the problem, we will always be able to pair two regions with an odd number of zeros to create the regions with the even number of zeros.

Outline of the construction of V. The case when the regions avoid the positive imaginary axis $i\mathbb{R}_+$ was handled correctly in the paper [**3**]. We now describe the situations when the regions intersect the positive imaginary axis $i\mathbb{R}_+$ and the construction necessary. In this case we need to pair regions $\mathcal{R}$ and $\mathcal{R}'$ together in an appropriate way. Similarly we need to pair to real zeros from σ_1. We additionally will need to join a region $\mathcal{R}$ and a zero from σ_1. We discuss each of these constructions now.

Connecting regions $\mathcal{R}$ and $\mathcal{R}'$ with an "odd" number of zeros. We first consider the case of when there are two symmetric regions $\mathcal{R}$ and $\mathcal{R}'$ that intersect $i\mathbb{R}_+$ and there are an *even* number of real zeros of p in $\mathcal{R}\cap\mathcal{R}'\cap i\mathbb{R}_+$. Let $B_{\mathcal{R}} := \prod_{a\in\sigma\cap\mathcal{R}} b_a$. Now note that the function $B_{\mathcal{R}}B_{\mathcal{R}'}\in H^\infty_{\mathbb{R}}(\mathbb{C}_+)$ since the domains $\mathcal{R}$ and $\mathcal{R}'$ are symmetric. We also have that the function $B_{\mathcal{R}}B_{\mathcal{R}'}$ is positive on $i\mathbb{R}_+$ above the domain $\mathcal{R}$ and below the domain $\mathcal{R}'$. We then connect these two domains via a slit I on the positive imaginary axis $i\mathbb{R}_+$. We can then take all the corresponding slits from the region $\mathcal{R}$, except the slit on the imaginary axis connecting $\mathcal{R}$ and $\mathbb{R}$. We can also then take the slits for the domain $\mathcal{R}'$ and this decomposes the domain $\mathbb{C}_+\setminus(\mathcal{R}\cup\mathcal{R}'\cup I)$ into connected components. By construction of the slits and the domains $\mathcal{R}$ and $\mathcal{R}'$ we have that the slits are disjoint, and similarly the δ' neighborhoods of the slits will be disjoint. We are now in the setup where we have a symmetric region with an even number of zeros and so the discussion above holds.

We then define,

$$\varphi(z) := \begin{cases} 0 & : \ z\in\mathcal{R}_{\delta'}\cup\mathcal{R}'_{\delta'}\cup I_{\delta'} \\ \log(B_{\mathcal{R}}B_{\mathcal{R}'}) & : \ \mathbb{C}_+\setminus(\mathcal{R}_{\delta'}\cup\mathcal{R}'_{\delta'}\cup I_{\delta'})\,, \end{cases}$$

for an appropriate branch of the logarithm that is real symmetric. Repeating the argument from above, it is possible to then split the domain $\mathbb{C}_+\setminus(\mathcal{R}_{\delta'}\cup\mathcal{R}'_{\delta'}\cup I_{\delta'})$ into connected components via slits and Γ-slits, and enlarge the slits by hyperbolic neighborhoods so that they are disjoint. We can then define in each such domain a branch of logarithm that is analytic, bounded and real symmetric since $B_{\mathcal{R}}B_{\mathcal{R}'}$ has an even number of zeros on $i\mathbb{R}_+$. The construction is explained in the figure below.

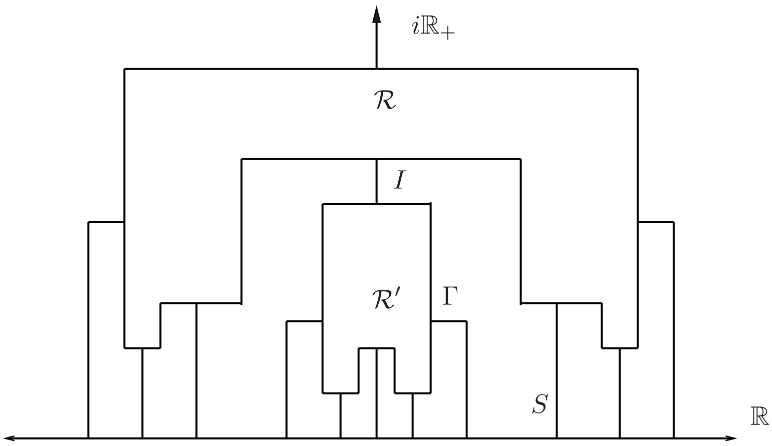

Figure 1

As before in [**3**], let $\mathcal{S}$ denote the collection of all slits for the component $\mathcal{R}$ and $\mathcal{R}'$ and the slit joining these regions together (except the slit connecting $\mathcal{R}$ to $\mathbb{R}$) and let $\Gamma^{\delta'}_{\mathcal{R},\mathcal{R}'}$ denote the $\frac{\delta'}{100}$ hyperbolic neighborhood of the region $\mathcal{R}\cup\mathcal{R}'\cup I$. These estimates and the boundedness of the jumps of φ allow one to change the function φ on the set $\Gamma^{\delta'}_{\mathcal{R},\mathcal{R}'}\cup\bigcup_{S\in\mathcal{S}} S_{\delta'}$ to obtain a function $V_{\mathcal{R},\mathcal{R}'}$ satisfying

$$V_{\mathcal{R},\mathcal{R}'} = \varphi(z) \quad \forall\, z \notin \Gamma^{\delta'}_{\mathcal{R},\mathcal{R}'} \cup \left(\bigcup_{S\in\mathcal{S}} S_{\delta'}\right),$$

$$|V'_{\mathcal{R},\mathcal{R}'}(z)| \le \frac{C(\delta,\epsilon)}{\operatorname{Im} z}, \quad |\Delta V_{\mathcal{R},\mathcal{R}'}(z)| \le \frac{C(\delta,\epsilon)}{(\operatorname{Im} z)^2} \quad \forall\, z \in \Gamma^{\delta'}_{\mathcal{R},\mathcal{R}'},$$

$$|V'_{\mathcal{R},\mathcal{R}'}(z)| \le \frac{C(\delta,\epsilon)}{d}, \quad |\Delta V_{\mathcal{R},\mathcal{R}'}(z)| \le \frac{C(\delta,\epsilon)}{d^2} \quad \forall\, z \in S_{\delta'}.$$

As before, to obtain the desired function $V_{\mathcal{R},\mathcal{R}'}$, simply convolve the function φ with an appropriate symmetric smooth kernel. Here the parameter d is the altitude of the slit S as defined in [**2**] and [**3**].

Connecting two regions corresponding to zeros in $\boldsymbol{\sigma_1}$. The next case arises when we have to connect two zeros in σ_1 on the imaginary axis. Let a, a' denote the zeros labeled so that a is the point closest to the real axis. About a let D_a denote the disc with center at the point a of radius $\delta' \operatorname{Im} a$, and let $T_a = \partial D_a$. Repeat this construction for the point a'. Then connect these discs together by drawing a vertical slit on the imaginary axis, denoted $I_{a,a'}$, which connects D_a to $D_{a'}$. Finally, draw the slit $I_{0,a}$ connecting the boundary of the disc D_a with the real axis. The construction is explained in the figure below.

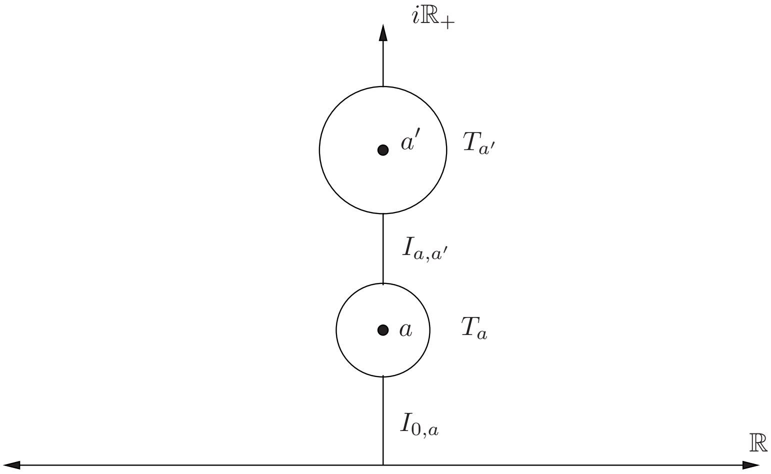

FIGURE 2

Then, one notes that $|b_a b_{a'}| \leq C(\delta, \epsilon)$ on $D_a \cup D_{a'} \cup I_{a,a'}$ and $|b_a b_{a'}| \geq C(\delta, \epsilon)$ for points in the complement. We then define φ in the following manner,

$$\varphi(z) := \begin{cases} 0 & : \ z \in D_a \cup D_{a'} \cup I_{a,a'} \\ \log(b_a b_{a'}) & : \ \text{otherwise.} \end{cases}$$

Note that we have $b_a b_{a'}$ is real and positive on $i\mathbb{R}_+$ above the point $a' + \delta' \operatorname{Im} a'$ and below $a - \delta' \operatorname{Im} a$. Clearly we have that φ is analytic, bounded (with a bound depending on δ and ϵ) and real symmetric. We again smooth φ to find our function V. Namely, we change φ in a $\frac{\delta'}{100} \min\{\operatorname{Im} a, \operatorname{Im} a'\}$ neighborhood of $I_{0,a} \cup T_a \cup I_{a,a'} \cup T_{a'}$ to obtain a smooth function on $\mathbb{C}_+$ such that:

(i) $|\bar{\partial} V_{a,a'}(z)| \leq C(\delta, \epsilon) \min\left\{\frac{1}{\operatorname{Im} a'}, \frac{1}{\operatorname{Im} a}\right\}$;

(ii) $|\Delta V_{a,a'}(z)| \leq C(\delta, \epsilon) \min\left\{\frac{1}{(\operatorname{Im} a')^2}, \frac{1}{(\operatorname{Im} a)^2}\right\}$;

(iii) $V_{a_1,a_2}(z) = \varphi(z)$ if $\operatorname{dist}(z, T_a \cup I_{a,a'} \cup T_{a'}) > \frac{\delta'}{100} \min\{\operatorname{Im} a, \operatorname{Im} a'\}$;

(iv) $V_{a,a'}(z) = \overline{V_{a,a'}(-\overline{z})}$.

Connecting a region $\mathcal{R}$ with a zero in σ_1. The final case is when we have to pair a region $\mathcal{R}$ with an odd number of zeros in $i\mathbb{R}_+$ and a zero $a \in \sigma_1 \cap i\mathbb{R}_+$. We describe the case when the zero in $\sigma_1 \cap i\mathbb{R}_+$ lies below the domain $\mathcal{R}$ since the opposite situation is similar. We connect the region $\mathcal{R}_{\delta'}$ and the region D_a by a slit $I_{\mathcal{R},a}$ on the imaginary axis. We then take the slits corresponding to the region $\mathcal{R}$, except for the slit on the imaginary axis connecting the region $\mathcal{R}$ with $\mathbb{R}$. We finally connect the boundary of D_a to the real axis by a slit I on the imaginary axis.

Now consider the function $B_{\mathcal{R}} b_a \in H^\infty_{\mathbb{R}}(\mathbb{C}_+)$ which has an even number of real zeros, and so, when we consider the simply connected regions given by the splitting of $\mathbb{C}_+ \setminus (\mathcal{R}_{\delta'} \cup D_a \cup I_{\delta'})$ using the appropriate slits, we can define a real symmetric branch of the logarithm.

In this situation, we now define

$$\varphi(z) := \begin{cases} 0 & : \ z \in \mathcal{R}_{\delta'} \cup D_a \cup I_{\delta'} \\ \log(B_{\mathcal{R}} b_a) & : \ \mathbb{C}_+ \setminus (\mathcal{R}_{\delta'} \cup D_a \cup I_{\delta'})\,, \end{cases}$$

where we choose a branch of the logarithm that is real symmetric. A repetition of the argument from above, shows that it is possible to then split the domain $\mathbb{C}_+ \setminus (\mathcal{R}_{\delta'} \cup D_a \cup I_{\delta'})$ into connected components via slits and Γ-slits, and enlarge the slits by hyperbolic neighborhoods so that they remain disjoint. We can then define in each such domain a branch of logarithm that is analytic, bounded and real symmetric since $B_{\mathcal{R}} b_a$ has an even number of zeros on $i\mathbb{R}_+$.

As above we let $\mathcal{S}$ denotes the collection of all slits for the component $\mathcal{R}$ (with out the slit joining $\mathcal{R}$ to $\mathbb{R}$) and the slit joining $\mathcal{R}$ and D_a and the slit joining together D_a and $\mathbb{R}$. Let $\Gamma^{\delta'}_{\mathcal{R}}$ denote the $\frac{\delta'}{100}$ hyperbolic neighborhood of the region $\mathcal{R}$. Let $T^{\delta'}_a$ denote the $\delta' \operatorname{Im} a$ neighborhood of T_a. The estimates from above and the boundedness of the jumps of φ allow one to change the function φ on the set $\Gamma^{\delta'}_{\mathcal{R},\mathcal{R}'} \cup \bigcup_{S\in\mathcal{S}} S_{\delta'} \cup T^{\delta'}_a$ to obtain a function $V_{\mathcal{R},a}$ satisfying

$$V_{\mathcal{R},a} = \varphi(z) \quad \forall\, z \notin \Gamma^{\delta'}_{\mathcal{R}} \cup \left(\bigcup_{S\in\mathcal{S}} S_{\delta'} \right) \cup T^{\delta'}_a,$$

$$|V'_{\mathcal{R},a}(z)| \leq \frac{C(\delta,\epsilon)}{\operatorname{Im} z}, \quad |\Delta V_{\mathcal{R},a}(z)| \leq \frac{C(\delta,\epsilon)}{(\operatorname{Im} z)^2} \quad \forall\, z \in \Gamma^{\delta'}_{\mathcal{R}},$$

$$|V'_{\mathcal{R},a}(z)| \leq \frac{C(\delta,\epsilon)}{d}, \quad |\Delta V_{\mathcal{R},a}(z)| \leq \frac{C(\delta,\epsilon)}{d^2} \quad \forall\, z \in S_{\delta'}.$$

Convolution of φ produces the smooth function $V_{\mathcal{R},a}$ that we seek.

Demonstrating the properties of V. It then remains to demonstrate the the function V has all the desired properties in Proposition 2.2. However, the general argument in [**3**] handles this. One needs only incorporate the changes in the construction of the function V, but with this new construction the symmetry has been preserved and all the claimed properties hold. For the sake of brevity of this corrigenda we omit the details, though point the reader to the `http://arxiv.org/abs/0809.1573`.

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