The evolution problem associated with eigenvalues of the Hessian

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Equations involving the eigenvalues of the Hessian matrix D^2u include some famous ones, like the Laplacian $\Delta u = \lambda_1 + ... + \lambda_N$, Monge-Ampere, $\det(D^2u) = \lambda_1 \cdots \lambda_N$, and λ_1 that is associated with the convex envelope.

In this talk we deal with the evolution problem

$$\begin{cases} u_t(x,t) - \lambda_j(D^2u(x,t)) = 0, & \text{in } \Omega \times (0,+\infty), \\ u(x,t) = g(x,t), & \text{on } \partial\Omega \times (0,+\infty), \\ u(x,0) = u_0(x), & \text{in } \Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ (that verifies a suitable geometric condition on its boundary) and $\lambda_j(D^2u)$ stands for the $j-$st eigenvalue of the Hessian matrix D^2u. We assume that u_0 and g are continuous functions with the compatibility condition $u_0(x) = g(x,0)$, $x \in \partial\Omega$.

We show that the (unique) solution to this problem exists in the viscosity sense and can be approximated by the value function of a two-player zero-sum game as the parameter measuring the size of the step that we move in each round of the game goes to zero.

In addition, when the boundary datum is independent of time, $g(x,t) = g(x)$, we show that viscosity solutions to this evolution problem stabilize and converge exponentially fast to the unique stationary solution as $t \to \infty$. For $j = 1$ the limit profile is just the convex envelope inside Ω of the boundary datum g, while for $j = N$ it is the concave envelope. We obtain this result with two different techniques: with PDE tools and and with game theoretical arguments. Moreover, in some special cases (for affine boundary data) we can show that solutions coincide with the stationary solution in finite time (that depends only on Ω and not on the initial condition u_0).

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