Blow up dynamics
for the hyperbolic vanishing mean curvature flow
of surfaces asymptotic to Simons cone

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We consider the hyperbolic vanishing mean curvature flow of surfaces in $\mathbb{R}^8$ asymptotic at infinity to Simons cone:

$$C_4 = \Big\{ X = (x_1, \cdots, x_8) \in \mathbb{R}^8 \,, x_1^2 + \cdots + x_4^2 = x_5^2 + \cdots + x_8^2 \Big\} \,.$$

We show that the flow admits finite time blow up solutions $(\Gamma(t))_{0<t\leq T}$ that blow up by concentration of the stationary profile: there exists a smooth minimal surface M asymptotic at infinity to Simons cone such that

$$\Gamma(t) \sim t^{\nu+1} \, M \,, \ \text{as } t \to 0 \,,$$

uniformly on compact sets, where ν is an arbitrary large positive number.

This issue amounts to investigate the singularity formation for a second order quasilinear wave equation. Our constructive approach consists in proving the existence of finite time blow up solutions of this hyperbolic equation under the form $u(t,x) \sim t^{\nu+1} Q\Big(\frac{x}{t^{\nu+1}}\Big)$, where Q is a stationary solution. Our strategy roughly follows that of Krieger, Schlag and Tataru initiated for the energy critical focusing semilinear wave equation. The goal of our result is to show that this blow up mechanism exists as well for the quasilinear wave equation.

This is a joint work with Alaa Marachli and Galina Perelman from Université Paris-Est Créteil.