Gaffney inequality:
best constant problem and some generalizations

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Gaffney inequality, stated here for vector fields in $\mathbb{R}^3$ for simplicity, says that

$$\|\nabla\omega\|^2_{L^2(\Omega)} \leq C_\Omega \left(\|\operatorname{curl}\omega\|_{L^2(\Omega)} + \|\operatorname{div}\omega\|_{L^2(\Omega)} + \|\omega\|_{L^2(\Omega)}\right)$$

for all vector fields ω whose tangential or normal part vanishes on $\partial\Omega$. The importance of Gaffney inequality arises from the Helmoltz/Hodge-Morrey decomposition with boundary conditions and some other boundary value problems in fluid dynamics or electromagnetism. In this talk I will mainly concentrate on the best constant problem, which deals with geometric properties of the domain, respectively generalizations of the boundary conditions. The talk will be partially based on some recent work with other collaborators taken from the following references:

– Csató G., Kneuss O. and Rajendran D., On the boundary conditions in estimating $\nabla\omega$ by $\operatorname{div}\omega$ and $\operatorname{curl}\omega$, *Proc. Roy. Soc. Edinburgh Sect. A*, to appear.

– Csató G., Dacorogna B. and Sil S., On the best constant in Gaffney inequality, *J. Funct. Anal.*, 274 (2018), no. 2, 461–503.