Transport equations and continuity equations with quasiconformal flows

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The transport equation

$$\begin{cases} \dfrac{d}{dt}\, u + b \cdot \nabla u = 0 \\ u(0,\cdot) = u_0 \end{cases}$$

and its adjoint, the continuity equation,

$$\begin{cases} \dfrac{d}{dt}\, \rho + \operatorname{div}(b\,\rho) = 0 \\ \rho(0,\cdot) = \rho_0 \end{cases}$$

appear as the result of linearization in a number of PDE's coming from fluid mechanics (Euler equation, surface quasigeostrophic equation, aggregation equation). The regularity of the initial data u_0 and ρ_0 is expected to be preserved as soon as the vector field b meets suitable properties (for instance, $Db \in L^\infty$).

In this talk we study the well-posedness of the Cauchy problem for the transport equation in the BMO space and for the continuity equation in the Hardy space H^1. In both cases, the existence of a unique flow of quasiconformal mappings $\phi_t(x)$ satisfying

$$\frac{d}{dt}\, \phi_t(x) = b(t, \phi_t(x)), \qquad \phi_0(x) = x.$$

plays a fundamental role.

This talk is based on the works:

Clop, Albert; Jiang, Renjin; Mateu, Joan; Orobitg, Joan. *Linear transport equations for vector fields with subexponentially integrable divergence.* Calc. Var. Partial Differential Equations 55 (2016), no. 1, Art. 21, 30 pp.

Clop, Albert; Jiang, Renjin; Mateu, Joan; Orobitg, Joan. *Flows for non-smooth vector fields with subexponentially integrable divergence.* J. Differential Equations 261 (2016), no. 2, 1237–1263.

Clop, Albert; Jiang, Renjin; Mateu, Joan; Orobitg, Joan. *A note on transport equation in quasiconformally invariant spaces.* Adv. Calc. Var. 11 (2018), no. 2, 193–202.

Clop, Albert; Jylhä, Heikki; Mateu, Joan; Orobitg, Joan. *Well-posedness for the continuity equation for vector fields with suitable modulus of continuity.* arXiv:1701.04603