VARIATIONAL TECHNIQUES IN GEOMETRIC FUNCTION THEORY, NONLINEAR PDES AND HYPERELASTICITY

Lectures in UAB by Tadeusz Iwaniec (Syracuse University), July 16-20, 2018

LECTURE 1 (Monday, July 16)

DIRECT METHOD IN THE CALCULUS OF VARIATIONS

- Brief historical account
- Harmonic Dirichlet Problem
- p-harmonic energy
- Convex functionals
- Traction free problems
- Abstract formulation of the direct method
- Weak continuity of Jacobian determinants
- Null-Lagrangians
- Polyconvex energy
- Morrey's concept of quasiconvexity

LECTURE 2 (Tuesday, July 17)

JACOBIAN DETERMINANTS AND SUBDETERMINANTS

- Distributional Jacobian
- Div-Curl Lemma
- L(log L) - integrability of nonnegative Jacobian (S. Müller)
- Jacobian in the Hardy space $\mathscr{H}^1(\mathbb{R}^n)$ (R. Coifman, P.-L. Lions, Y. Meyer, and S. Semmes)
- $\mathscr{H}^1$-estimates of Jacobians in terms of cofactors (T. Iwaniec, and J. Onninen)
- Remarks on Compensated Compactness

LECTURE 3 (Wednesday, July 18)

FREE LAGRANGIANS

- Definition of free Lagrangians
- Some differential forms
- Examples of free Lagrangians
- Applications of free Lagrangians
- Energy-Minimal mappings between annuli

- Deformations of 2D-domains (plates) and surfaces (thin films)
- Mappings of smallest mean distortion ($\mathscr{L}^p$-Teichmüller mappings)

LECTURE 4 (Thursday, July 19)

DIFFEOMORPHIC APPROXIMATION OF SOBOLEV MAPPINGS

- J. Ball-C. Evans Conjecture
- Approximation of a Sobolev homeomorphism with diffeomorphisms
- Radó-Kneser-Choquet Theorem
- Weak and strong limits of Sobolev homeomorphisms
- Diffeomorphic approximation of monotone Sobolev mappings
- Triangulation of diffeomorphisms
- Existence of energy-minimal deformations within Sobolev monotone mappings, and the Principle of Non-Interpenetration of Matter
- Hopf differential and formation of cracks

LECTURE 5 (Friday, July 20)

THE NITSCHE CONJECTURE

- Schottky Theorem
- Harmonic mappings
- Nitsche Conjecture (historical remarks)
- Minimal surfaces
- Sketch of the proof