Magnetic Helicity and Subsolutions in MHD

Daniel Faraco
Universidad Autónoma de Madrid

In the recent years, the world of hydrodynamics have experienced a revolution due to the new method of building weak solutions created by De Lellis and Szekelyhidi for the Euler equation based on convex integration. Such solutions seems to be the best currently available model to explain the phenomena anomolous disspation of several quantities in turbulent regimes.The method yields bizzard solutions which have compact support in space and time, violating severely uniqueness of the Cauchy problem.

In this talk, I will describe how this circle of ideas interplay with Magneto Hydrodynamics, in short MHD, that is Maxwell and Euler equations coupled. At first glance, one is tempted to to believe that the results for Euler could be replicated with lenghtier and more complex computations. However, I will present a proof showing that such compactly supported solutions do not exist in 2 D MHD. In 3D, we can show that such compacty supported subsolution exists but there is an integral quantity, magnetic helicity which is preserved even for irregular solutions. We will show that it is also preserved by subsolutions and explain that has an interpretation as a compensated compactness quantity which imposes a nonlinear constraint on the space of subsolutions. This is a joint work with Sauli Lindberg.