Removability, capacity and approximation

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Abstract

In this paper we are primarily interested in problems of qualitative approximation by holomorphic functions of one complex variable belonging to some fixed class, that is defined by restricting the growth of the functions (L^p, $1 < p \leq \infty$) or by requiring certain smoothness (Lip s or C^m). Part of the approximation problem consists in understanding the removable sets for the class under consideration and its associated capacity.

In Chapter 1 we deal with bounded analytic functions. We are thus led to the Painlevé problem and analytic capacity. We discuss the solution of the Denjoy conjecture via L^2-estimates for the Cauchy integral on Lipschitz graphs. We then show that the same ideas can be applied to describe removable sets for Lipschitz analytic functions, the role of the Cauchy integral being played by the Beurling transform.

Chapter 2 is devoted to Vitushkin's Theorem on uniform approximation by rational functions; the simplest available proof is described in detail.

In Chapter 3, problems of approximation by analytic functions in Lipschitz and C^m classes are considered. Vitushkin's scheme and the mapping properties of the Beurling transform are combined to obtain satisfactory answers to the main questions.

In Chapter 4 we discuss the relationship between L^p-approximation by analytic functions and spectral synthesis for Sobolev spaces.

Chapter 5 is a survey of recent results about approximation by solutions of elliptic equations in classical Banach spaces.

Contents

P. M. Gauthier (ed.) and G. Sabidussi (techn. ed.), Complex Potential Theory, 419–473.

Introduction

This paper grew out of a series of lectures given by the author at the 1993 Montreal Summer School on Complex Potential Theory.

Our starting point is the notion of set of removable singularities for functions satisfying a given partial differential equation, and subject to some previously specified growth