

Abel quadratic differential systems of second kind

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Abstract

The Abel differential equations of second kind, named after Niels Henrik Abel, are a class of ordinary differential equations studied by many authors. Here we consider the Abel quadratic polynomial differential equations of second kind denoting this class by QS_{Ab} . Firstly we split the whole family of non-degenerate quadratic systems in four subfamilies according to the number of infinite singularities. Secondly for each one of these four subfamilies we determine necessary and sufficient affine invariant conditions for a quadratic system in this subfamily to belong to the class QS_{Ab} . Thirdly we classify all the phase portraits in the Poincaré disc of the systems in QS_{Ab} in the case when they have at infinity either one triple singularity (21 phase portraits) or an infinite number of singularities (9 phase portraits). Moreover we determine the affine invariant criteria for the realization of each one of the 30 topologically distinct phase portraits.

Key-words: quadratic differential system, second kind of Abel differential equations, phase portraits.

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1 Introduction and statement of the main results

We consider the class of real quadratic polynomial differential systems

$$\begin{aligned}\dot{x} &= p_0 + p_1(x, y) + p_2(x, y) \equiv P(\tilde{a}, x, y), \\ \dot{y} &= q_0 + q_1(x, y) + q_2(x, y) \equiv Q(\tilde{a}, x, y)\end{aligned}\tag{1}$$

where

$$\begin{aligned}p_0 &= a, & p_1(x, y) &= cx + dy, & p_2(x, y) &= gx^2 + 2hxy + ky^2, \\ q_0 &= b, & q_1(x, y) &= ex + fy, & q_2(x, y) &= lx^2 + 2mxy + ny^2.\end{aligned}$$