WHEN SINGULAR POINTS DETERMINE QUADRATIC SYSTEMS

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Abstract. When one considers a quadratic differential system, one realizes that it depends on 12 parameters of which one can be fixed by means of a time change. One also can notice that fixing 4 finite real singular points plus 3 infinite real ones (all its possible singular points) implies to fix 11 conditions, that is, 11 equations that the parameters must satisfy. Since these conditions are linear with respect to the parameters, it is obvious to think that the system will be determined, except that the fixed conditions are incompatible with a quadratic differential system having finitely many singular points.

In this paper we prove exactly this. That is, if we fix the position of the 7 singular points of a quadratic differential system in a distribution that does not force an infinite number of finite singular points, then the system is completely determined, and consequently its phase portrait is also determined. This determination includes the local behavior of all singular points, even if they are weak focus or centers, the global behavior of separatrices, and even the existence or not of limit cycles. This also implies that limit cycles are sensitive to small perturbations of the coordinates of singular points, even if they are far from the singular points.

The result of the paper goes far beyond this, since we state that this result is independent of the fact that the fixed singular points are real or complex, and it does not mind if the infinite singular points are simple or multiple due to the collision of several infinite singular points. Only when some data is lost due to the collision of finite singular points or to the collision of some finite singular points with infinite ones, this adds free parameters to the set of parameters at the same rate than the number of finite singular points are lost.

1. Introduction and statement of the main results

We consider the real polynomial differential systems

s:gen (1) $$\frac{dx}{dt} = P(x,y), \qquad \frac{dy}{dt} = Q(x,y),$$

where P and Q are polynomials in the variables x and y with real coefficients, i.e. $P,\ Q \in \mathbb{R}[x,y]$. We shall say that systems (1) are *quadratic* if $\max(\deg(P), \deg(Q)) = 2$.

Quadratic differential systems have been studied from many different points of view (the following lists clearly are not complete): studying their finite singular points [12, 13, 9, 10, 22, 23, 24, 51, 7], studying their infinite singular points [30, 33, 48], studying systems with limit cycles [18, 17], studying systems with invariant straight lines [37, 38, 39, 47, 46], studying systems with centers [25, 29, 26, 44, 15, 51, 45, 52], and systems with weak focus [5, 32, 6], studying systems with invariant algebraic curves or first integrals [4, 31], classifying phase portraits according to the number of finite singular points [27, 41, 42, 43], classifying phase portraits according to the structural stability

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