A COMPLETE CLASSIFICATION OF DEGENERATE QUADRATIC DIFFERENTIAL SYSTEMS

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Abstract. In this paper we present a complete study of degenerate quadratic differential systems, i.e. the polynomials from right-hand parts of these systems are not co-prime. We give the complete set of the phase portraits. as well necessary and sufficient conditions for the realization of each one of them. These conditions are constructed in invariant form and we present here the "bifurcation" diagram directly in the space $\mathbb{R}^{12}$ of the whole set of the quadratic systems.

This paper is part of a project which ultimate goal is the complete classification of topologically distinct phase portraits of quadratic systems modulo limit cycles. We also provide a label for each phase portrait inside the global codification related to the global configurations of singularities and their codimensions.

1. Introduction and the statement of the main result

We consider here differential systems of the form

$$\frac{dx}{dt} = p(x, y), \qquad \frac{dy}{dt} = q(x, y), \tag{1}$$

where p, $q \in \mathbb{R}[x, y]$, i.e. p, q are polynomials with real coefficients in x, y. We call *degree* of a system (1) the integer $n = \max(\deg p, \deg q)$. In particular we call *quadratic* a differential system (1) with $n = 2$. We denote here by **QS** the whole class of real quadratic differential systems.

The motivation of the authors for writing this paper is to advance towards the complete classification of topologically distinct phase portraits of quadratic differential systems modulo limit cycles. During the last decades, more that one thousand of papers have been dedicated to quadratic systems producing many hundreds of phase portraits.

The approaches used by these papers have been of different kinds:

- some papers considered phase portraits of quadratic systems under very restricted conditions like: existence of centers (see [38, 43, 48]), chordal quadratic systems (see [24, 35]), systems with just one finite singularity (see [17, 36]) and many others;
- some papers considered phase portraits of a large family of quadratic systems with few restrictions obtaining the full corresponding bifurcations diagrams like: systems with a finite nilpotent singularity [29], systems with a weak focus of order two [4], systems with two saddle-nodes [11, 12, 15] and many others;
- some papers considered phase portraits of the most generic classes like: structurally stable phase portraits of quadratic systems (i.e. codimension 0) [2], structurally unstable phase portraits of quadratic systems of codimension 1 [3] and some papers dealing with phase portraits of codimension 2 [1, 13, 14].

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