SINGULAR POINTS OF QUADRATIC SYSTEMS: A COMPLETE CLASSIFICATION IN THE COEFFICIENT SPACE $\mathbb{R}^{12}$

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Planar quadratic differential systems occur in many areas of applied mathematics. Although more than one thousand papers were written on these systems, a complete understanding of this class is still missing. Classical problems, and in particular, Hilbert's 16th problem [Hilbert, 1900], are still open for this class. Even when not dealing with limit cycles, still some problems remain unsolved like a complete classification of different phase portraits without limit cycles.
For some time it was thought (see [Coppel, 1966]) that it could exist a set of algebraic functions whose signs would completely determine the phase portrait of a quadratic system. Nowadays we already know that this is not so, and that there are some analytical, non–algebraic functions that play also a role when dealing with limit cycles and separatrix connections.
However, it is possible to find out a set of algebraic functions whose signs determine the characteristics of all finite and infinite singular points. Most of the work up to now has dealt with this problem studying it using different normal forms adapted to some subclasses of quadratic systems. A general work useful for any quadratic system regardless of affine changes has only been done for the study of infinite singular points [Schlomiuk *et al.*, 2005].
In this article we give a complete global classification of quadratic differential systems according to their topological behavior in the vicinity of the finite singular points. Our classification Main Theorem gives us a complete dictionary describing the local behavior of finite singular points using algebraic invariants and comitants which are a powerful tool for algebraic computations. Linking the result of this article with the main one of [Schlomiuk *et al.*, 2005] which uses the same algebraic invariants it is possible to complete the algebraic classification of singular points (finite and infinite) for quadratic differential systems.