CONFIGURATIONS OF SINGULARITIES FOR QUADRATIC DIFFERENTIAL SYSTEMS WITH THREE FINITE REAL DISTINCT SINGULARITIES

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Abstract. ??? In this work we consider the problem of classifying all configurations of singularities, both finite and infinite of quadratic differential systems, with respect to the *geometric equivalence relation* defined in [3]. This relation is finer than the *topological equivalence relation* which does not distinguish between a focus and a node or between a strong and a weak focus or between foci of different orders. Such distinctions are however important in the production of limit cycles close to the foci in perturbations of the systems. The notion of *geometric equivalence relation* of configurations of singularities allows to incorporates all these important geometric features which can be expressed in purely algebraic terms. This equivalence relation is also finer than the *qualitative equivalence relation* introduced in [17]. The *geometric classification* of all configurations of singularities, finite and infinite, of quadratic systems was initiated in [4] where the classification was done for systems with total multiplicity m_f of finite singularities less than or equal to one. In this article we continue the work initiated in [4] and obtain the *geometric classification* of singularities, finite and infinite, for the subclass of quadratic differential systems possessing finite singularities of total multiplicity $m_f = 2$. We obtain 197 *geometrically distinct* configurations of singularities for this family. We also give here the global bifurcation diagram of configurations of singularities, both finite and infinite, with respect to the *geometric equivalence relation*, for this class of systems. The bifurcation set of this diagram is algebraic. The bifurcation diagram is done in the 12-dimensional space of parameters and it is expressed in terms of polynomial invariants. The results can therefore be applied for any family of quadratic systems in this class, given in any normal form. Determining the geometric configurations of singularities for any such family, becomes thus a simple task using computer algebra calculations.

1. Introduction and statement of main results

We consider here differential systems of the form

$$\frac{dx}{dt} = p(x,y), \qquad \frac{dy}{dt} = q(x,y), \tag{1}$$

where $p,\ q \in \mathbb{R}[x,y]$, i.e. $p,\ q$ are polynomials in $x,\ y$ over $\mathbb{R}$. We call *degree* of a system (1) the integer $m = \max(\deg p, \deg q)$. In particular we call *quadratic* a differential system (1) with $m = 2$. We denote here by **QS** the whole class of real quadratic differential systems.

The study of the class **QS** has proved to be quite a challenge since hard problems formulated more than a century ago, are still open for this class. It is expected that we have a finite number of phase portraits in **QS**. Although we have phase portraits for several subclasses of **QS**, the complete list of phase portraits of this class is not known and attempting to topologically classify these systems, which occur rather often in applications, is a very complex task. This is partly due to the elusive nature of limit cycles and partly to the rather large number of parameters involved. This family of systems depends on twelve parameters but due to the group action of real affine transformations and time homotheties, the class ultimately depends on five

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