

Quadratic systems with a rational first integral of degree 2: a complete classification in the coefficient space \mathbb{R}^{12}

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Abstract

A quadratic polynomial differential system can be identified with a single point of \mathbb{R}^{12} through the coefficients. Using the algebraic invariant theory we classify all the quadratic polynomial differential systems of \mathbb{R}^{12} having a rational first integral of degree 2. We show that there are only 24 topologically different phase portraits in the Poincaré disc associated to this family of quadratic systems up to a reversal of the sense of their orbits, and we provide a unique representative of every class modulo an affine change of variables and a rescaling of the time variable. Moreover, each one of these 24 representatives is determined by a set of invariant conditions and each respective first integral is given in invariant form directly in \mathbb{R}^{12} .

1 Introduction

Let P and Q be two real polynomials in the variables x and y , then we say that

$$\dot{x} = \frac{dx}{dt} = P(x, y), \quad \dot{y} = \frac{dy}{dt} = Q(x, y),$$

is a *quadratic polynomial differential system* or simply a *quadratic system* if the maximum of the degrees of the polynomials P and Q is two. Associated to this quadratic system we

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