A UNIVERSAL CONSTANT FOR A SEMISTABLE LIMIT CYCLE

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Abstract. We restrict our attention to one–parameter families of polynomial vector fields X_b having a semistable limit cycle. We show numerically the existence of a kind of universal constant equal to one and of a power law of the form $b_n \approx Cn^\alpha$ for the parameter b associated to a semistable limit cycle. For this power there is numerical evidence that: (1) the exponent α of the power law depends only on the multiplicity of the semistable limit cycle, and it is independent of the polynomial vector field and its degree; (2) the constant C depends also on the way that we construct the sequence b_n.

1. Introduction and statement of the main result

By definition a *(planar) polynomial vector field* is a vector field of the form

$$X = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y},$$

where P and Q are polynomials in the real variables x and y. Usually and for simplicity we will denote the vector field X simply by (P,Q). The *degree* d of the polynomial vector field X is the maximum of the degrees of the polynomials P and Q.

For a definition of limit cycle, stable or unstable limit cycle, semistable limit cycle and multiplicity of a limit cycle, see for instance [17].

Let X_b be one–parameter family of polynomial vector fields depending on the parameter b. Suppose that X_0 has a semistable limit cycle Γ surrounding the origin O of $\mathbb{R}^2$, and that X_b for $b<0$ and small has no limit cycles in a given annular neighborhood N of Γ. Moreover, we assume that the flow of every X_b with $b<0$ enters into the annular

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