Tropical Elliptic Curves and their j-Invariant

(joint work with Eric Katz and Hannah Markwig)

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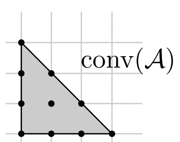
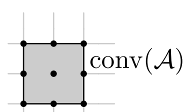
Examples



$X_{\mathcal{A}} \subset \mathbb{P}^9$

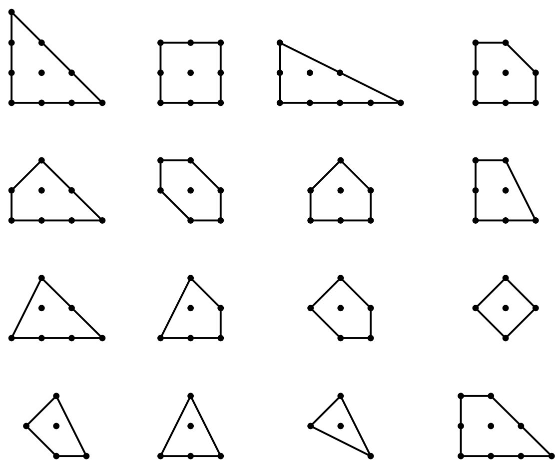
3-tuple Veronese embedding of $\mathbb{P}^2$



$X_{\mathcal{A}} \subset \mathbb{P}^8$

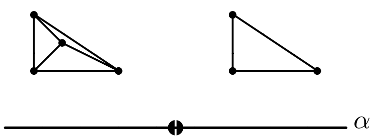
$(2, 2)$-Segre embedding of $\mathbb{P}^1 \times \mathbb{P}^1$

Classification of the polygons

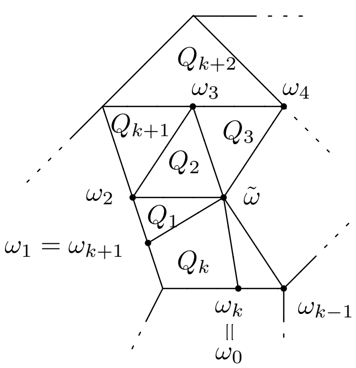


Possible Newton subdivisions

$$f = y^2 + x^3 + 1 + t^{\alpha} xy$$



Formula for j_{trop}



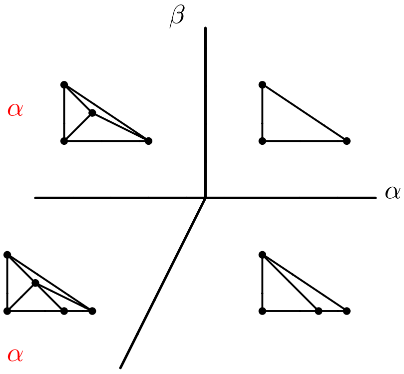
$$\sum_{j=1}^{k} (u_{\tilde{\omega}} - u_{\omega_j}) \cdot \frac{D_{j-1,j} + D_{j,j+1} + D_{j+1,j-1}}{D_{j-1,j} \cdot D_{j,j+1}}$$

where $D_{i,j} = \det(v_i, v_j)$ with $v_i = \omega_i - \tilde{\omega}$ and $v_j = \omega_j - \tilde{\omega}$.

Possible Newton subdivisions

$$f = y^2 + x^3 + 1 + t^\alpha xy + t^\beta x^2$$

$j_{\text{trop}} = -6 \cdot \alpha$



$j_{\text{trop}} = -6 \cdot \alpha$

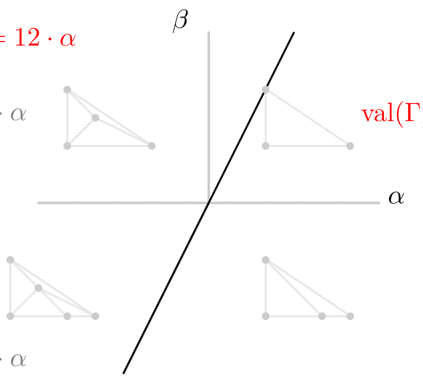
$$-\operatorname{val}(j) = \operatorname{val}(\Delta) - \operatorname{val}(\Gamma)$$

$\Gamma = (t^{2\alpha} - 4 \cdot t^{\beta})^6 \qquad \Delta = (t^{2\alpha} - 4t^{\beta})^3 - 432$

$\operatorname{val}(\Gamma) = 12 \cdot \alpha$

$j_{\text{trop}} = -6 \cdot \alpha$

$\operatorname{val}(\Gamma) = 6 \cdot \beta$



$j_{\text{trop}} = -6 \cdot \alpha$

$-\operatorname{val}(j) = \operatorname{val}(\Delta) - \operatorname{val}(\Gamma)$

$\Gamma = (t^{2\alpha} - 4 \cdot t^{\beta})^6 \qquad \Delta = (t^{2\alpha} - 4t^{\beta})^3 - 432$