Introduction to Surface Group Representations and Higgs Bundles
Assignment 6

Weizmann Institute
First Semester 2017-2018

There is no formal submission of the assignments but you must work on them.

One of the students will present a solution and we will discuss alternatives.

Problem 1. We have seen how to give a topology to the fundamental group of a topological space X. Give $\mathrm{Hom}(\mathrm{S}^1, X)$ the compact-open topology, consider the subspace $\mathrm{Hom}_x(\mathrm{S}^1, X)$ of loops that start and end at a fixed base point x, and finally consider the quotient topology when we quotient by the equivalence relation of fixed end point homotopy.

- Prove that for X a manifold we obtain the discrete topology.

Problem 2.

We have defined the universal cover $\bar{M}$ of a manifold M as equivalence classes of paths starting at a base point, where the equivalence is given by fixed end point homotopy.

- Prove that the universal cover has trivial fundamental group, or in other words, that it is simply connected.
- Try to find an action of $\pi_1 M$ on $\tilde{M}$ and describe the orbits.

Problem 3. Let $\Sigma_{g,k}$ be a compact connected orientable surface of genus g from which we remove k points. Recall how we computed the fundamental group of $\Sigma_{1,1}$, the torus $\Sigma_{1,0}$, and also $\Sigma_{g,1}$ and $\Sigma_{g,0}$.

- Use Seifert-Van Kampen theorem to give a presentation of the fundamental group of $\Sigma_{g,k}$.