Understanding and measuring rhythmic quality in dance. What is a movement accent? *

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Abstract

The concept of rhythmic quality in dance is analysed. Attention is focused on the simple case where the motion reduces to the translation of a single point. It is shown that the human perception of movement accents in such motions corresponds to defining these accents as strong concentrations in time of the magnitude of the acceleration vector. Practical methods for effectively measuring this variable are also studied.

1 Introduction

This article starts a project that aims at providing a method for measuring rhythmic quality in dance. This method is aimed especially at dancesport, where rhythm is supposed to be a foremost judging criterion. In spite of this importance, the existing dancesport technical specifications are not precise enough for allowing rhythmic quality to be measured. As a consequence, this concept is liable to subjective interpretation by the judges. This project aims at changing this state of affairs by showing that rhythmic quality is more objective than it seems.

For the moment, the project is aimed only at rhythmic quality in a strict sense, excluding any "expressive" deviations from the ideal pattern. On the other hand, the ideal pattern is not meant to be specific for a particular choreographic sequence, but it should be generic within a given dance genre.

The work to be developed in the project involves three main aspects: First, analysing the intended meaning of rhythmic quality according to dancesport experts and proposing a mathematical representation of it. Second, designing a practical method for effectively measuring it. Third, adjusting some parameters and checking that the results are in general agreement with the opinion of dancesport experts. This starting paper contributes mainly to the first of these three aspects.

There are several other works where dance rhythm is analysed by means of motion capture technology. In [3] some rhythm patterns are extracted from video signals by frame differencing and subsequent spectral analysis. In [1] pressure sensors are used to detect foot steps, which is suitable for teaching beginners, but not for assessing the rhythmic quality of developed dancing. Our work is more related to [6], where certain rhythm patterns are extracted by making use of accelerometer data. However, those authors leave the question of locating the movement accents largely unanswered (see [6, §4.1, under 'Timestamp']).

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2 Starting definitions

We intend to measure **rhythmic quality** as a *degree of agreement* between a performed *accentuation* and a specified *rhythm pattern.*

By an **accentuation** we mean simply a distribution in time of certain *accentuation variables.* Accordingly, we view an **accent** as a strong concentration of them at a particular location in time. A crucial issue to be adressed in this article is which are the accentuation variables relevant to dance.

A **rhythm pattern** is an ideal accentuation, usually one that repeats periodically in time. The **degree of agreement** could be measured by a sort of correlation integral between the performed accentuation and the ideal one. These aspects will be developed in another article.

Although dance is usually performed to music, the rhythm pattern for dance is not the music in itself. In fact, in principle one can look for a rhythm pattern in a motion without music, in the same way as one can look for it in a musical performance by itself. As a first approximation, one can think of dance and music as both of them having the same rhythm pattern. Generally speaking, however, their respective rhythm patterns need not be exactly the same, though they can be inferred from each other.

In the case of music, the notion of accent as a strong concentration of some measurable magnitude is suitable to the most obvious accents, namely the performed notes. In this connection, a good accentuation variable seems to be the logarithmic derivative of energy as measured from a localized spectral decomposition [2].

3 What is a movement accent?

3.1 In dance, there is a certain tendency to consider as rhythm-marking accents the steps of the feet. However, in dancesport and many other dance genres a good rhythmic quality lies more fundamentally in the movement of the centre of the body than in that of the feet. Generally speaking, however, the movement of the body centre is more flowing than the steps of the feet. So, the problem of identifying the rhythm-marking accents becomes a more difficult task.

The existing dance literature hardly goes into details about the precise meaning of a movement accent. Having said that, some authors make a try at it. For instance, Ann Hutchinson [5, p. 478 of 3rd ed.] defines a movement accent as "the result of a sudden momentary increase in the use of energy", and, more interestingly, she distinguishes the following classes of accents: "A strong accent at the start of a movement: *impulse*; a strong accent in the middle of a movement: often a *swing*; a strong accent at the end of a movement: *impact*". An impact merged with an immediately subsequent impulse makes a *rebound*. Besides these terms, there are other ones which have to do with sustained motion and the absence of an accent, or even the opposite of it: *vibration*, *steadiness* (motion without an accent), *stillness* (absence of motion), *suspension*. This language allows to describe movement accentuation in a qualitative way. For instance, Ruud Vermeij [8, p. 119–126] uses it to describe the main rhyhtmic character of each of the five latin dances of dancesport.

However, this is not enough for our purposes. If we are to measure the rhythmic quality of dance movement, we have to rely on some measurable accentuation variable. And it is not obvious which variable is the right one. In the words of Ruud Vermeij [8, p. 159]: "Even if we understand rhythm only as timing, the question remains, what is it that changes?"

3.2 We will base our quest on certain *empirical observations* about the human perception of rhythm in a motion. In this paper we will consider some very simple cases where the motion reduces to the translation of a single point (which one can think of as the centre of the body of a dancer). More specifically, we will look at the oscillations of springs and pendulums. Such oscillations are universally recognized as typical examples of rhythmic motions. So, it is natural to ask where does one locate their rhythm-marking accents. We have posed this question to a variety of people, most of them dancesport experts, and we have got the following answers:

EMPIRICAL OBSERVATION 1. *For the oscillations of a mass suspended by an ordinary spring, the movement accents are perceived at the return points.*

EMPIRICAL OBSERVATION 2. *For a ball that bounces on the floor, the movement accents are perceived at the bottom return points.*

EMPIRICAL OBSERVATION 3. *For a pendulum that oscillates with a small amplitude, the movement accents are perceived at the return points of both sides.*

EMPIRICAL OBSERVATION 4. *For a pendulum that oscillates with a large amplitude, the movement accents are perceived at the bottom.*

EMPIRICAL OBSERVATION 5. *For a pendulum that oscillates with an intermediate amplitude, there is a general hesitation or division of opinions between locating the movement accents at the return points of both sides, or locating them at the bottom.*

3.3 We claim that all of these facts are explained if one considers as accentuation variable the magnitude of the acceleration vector. In the following we will denote this magnitude as a. Our claim will be supported by an analysis based on the differential equations of these motions (see for instance [4]).

Both a mass suspended from a spring and a bouncing ball can be described by a differential equation of the form

$$\ddot{z} + f(z) = 0, \tag{1}$$

where z represents the vertical coordinate, every dot denotes a time derivative, and the function f has the following properties: (a) $f(z)$ vanishes only in the position of equilibrium $z = z_*$; (b) $(z - z_*) f(z) \geq 0$ for all z. Obviously, the magnitude of the acceleration is

$$a = |f(z)|. \tag{2}$$

By differentiating a^2 with respect to time, one obtains that

$$\frac{\mathrm{d}}{\mathrm{d}t} a^2 = 2 f(z) f'(z) \dot{z}, \tag{3}$$

from which one can find the extrema of a^2, and therefore those of a, as a function of t. If $f'(z) > 0$ for all z, as it is expected of the standard notion of a spring, then the set of critical points consists of two minima at the position of equilibrium $z = z_*$ (one for each direction of movement) and two maxima at the extreme values of z, that is, at the return points, in agreement with the empirical observation 1.

In the case of a bouncing ball, z represents the distance of the centre of the ball from the floor, and f is as follows: $f(z)$ equals the acceleration of gravity g whenever z is larger than the natural radius r of the ball; $f(z)$ quickly increases from $-\infty$ to g as z varies from ρ to r, where $\rho \geq 0$ is a lower limit for z. Such an f combines the force of gravity together with that made by the floor through the material structure of the ball. According to this model, the acceleration magnitude a is constantly equal to g while the ball is flying. So, there is not a strict maximum of a during the flight. In contrast, a exhibits a large and sharp strict maximum at the moment when z is minimum. This is quite in agreement with the empirical observation 2.

Let us consider now a friction-less pendulum under the action of gravity. Its motion is described by the differential equation

$$r\ddot{\varphi}/g + \sin\varphi = 0, \tag{4}$$

where φ represents the angle of departure from the vertical position (with the mass at the bottom), r is the length of the pendulum, and g is the acceleration of gravity. This equation has the property that energy is conserved, which fact can be expressed in the following adimensional form:

$$\tfrac{1}{2}\, r\,(\dot{\varphi})^2/g + 1 - \cos\varphi \,=\, \eta, \tag{5}$$

where η is constant over time. Different values of η correspond to different forms of motion: for $\eta = 0$ the motion reduces to staying at $\varphi = 0$; for $0 < \eta < 2$, the pendulum oscillates in the range $|\varphi| \le \arccos(1-\eta)$; for $\eta = 2$ the motion either reduces to staying at $\varphi = \pi$, which is an unstable equilibrium, or it tends to this equilibrium both as $t \to -\infty$ and as $t \to +\infty$; finally, for $\eta > 2$ complete-loop oscillations take place.

As it is well known, the acceleration vector has a tangential component of value $r\ddot{\varphi}$ and a centripetal normal component of magnitude $r(\dot{\varphi})^2$. Therefore, by Pythagoras' theorem,

$$a^2 \,=\, r^2(\ddot{\varphi})^2 + r^2(\dot{\varphi})^4.$$

By making use of equations (4) and (5), this expression leads to the following one:

$$a^2/g^2 \,=\, \sin^2\varphi + 4\,(\eta - 1 + \cos\varphi)^2. \tag{6}$$

By differentiating it with respect to time, one obtains that

$$\frac{\mathrm{d}}{\mathrm{d}t}\, a^2/g^2 \,=\, (4(1-\eta) - 3\cos\varphi)\ (\sin\varphi)\ \dot{\varphi}, \tag{7}$$

from which one can find the extrema of a^2/g^2 as a function of t. Table 1 classifies these critical points in four categories, which we have termed respectively *swing* (S), *rebound* (R), *suspension* (Z) and *medial* (M), and figures 1 and 2 show the graphs of $\eta \mapsto |\varphi|$ and $\eta \mapsto a/g$.

Name	*Value of* $\lvert\varphi\rvert$	*Interval of* η	*Value of* a/g
Swing (S)	0	$0 \le \eta$	2η
Rebound (R)	$\arccos(1-\eta)$	$0 \le \eta \le 2$	$(2\eta - \eta^2)^{1/2}$
Suspension (Z)	π	$2 \le \eta$	$2\eta - 4$
Medial (M)	$\arccos\left(\frac{4}{3}(1-\eta)\right)$	$\frac{1}{4} \le \eta \le 1$	$(1 - \frac{4}{3}(1-\eta)^2)^{1/2}$

Table 1. *Extrema of the acceleration magnitude for the pendulum.*

The solid curves of figures 1 and 2 correspond to local maxima of a whereas the dashed ones correspond to local minima. Therefore, defining a movement accent as a local maximum of a, has the following consequences: (a) the rebounds at both sides are the only accents for $\eta < \frac{1}{4}$ (small-amplitude oscillations); (b) the swings at the bottom (one in each direction) are the only accents for $\eta > 1$ (large-amplitude oscillations); and (c) both these kinds of critical points coexist as movement accents for $\frac{1}{4} < \eta < 1$, with the corresponding values of a crossing each other at $\eta = \frac{2}{5}$ (which corresponds to an amplitude of approximately 53°). These facts are quite in agreement with the empirical observations 3–5.

So, *the above-stated empirical facts are accounted for if one defines a movement accent as a maximum of the magnitude of the acceleration vector*.

Other candidates for accentuation variables —such as kinetic energy, its rate of change, single coordinates of the acceleration vector, or the magnitude of the sensed acceleration vector that we discuss in the next section— may happen to fit some particular perceived movement accents, but the magnitude of the acceleration vector is the only variable that we have found to explain all those empirical facts.

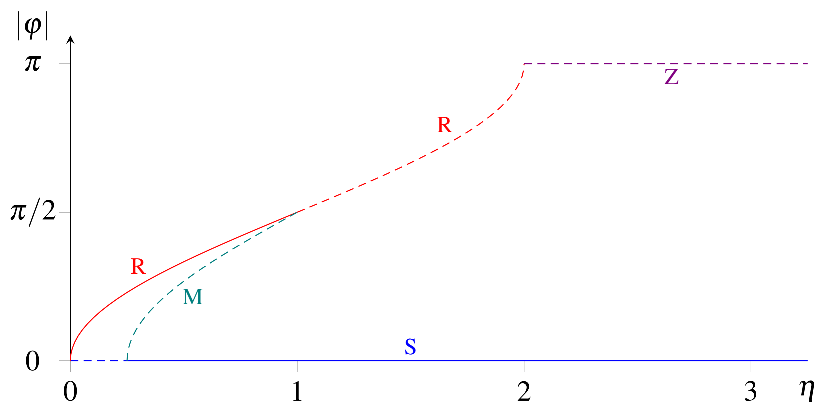

Figure 1. *Extrema of the acceleration magnitude for the pendulum. Graph of* $\eta \mapsto |\varphi|$.

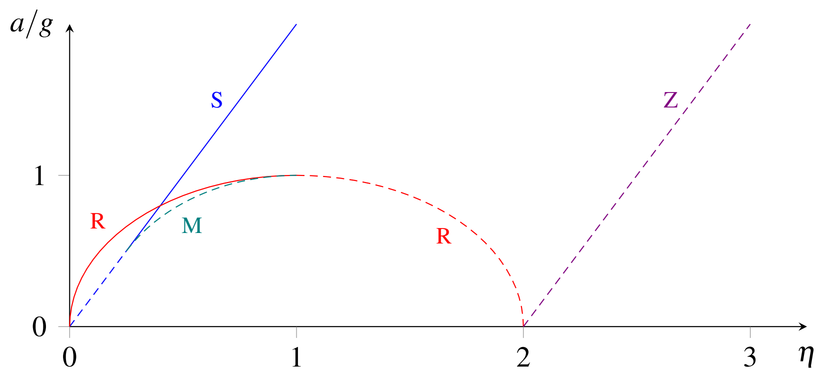

Figure 2. *Extrema of the acceleration magnitude for the pendulum. Graph of* $\eta \mapsto a/g$.

4 Towards measurement

The practical implementation of the preceding ideas requires a system for capturing the motion to be analysed. One way to do it is optical tracking (see for instance [7]). However, such systems are rather expensive and they present some technical difficulties for working out the acceleration from the measured position coordinates. Alternatively, one can use **inertial measurement units** based on microelectromechanical sensors (accelerometers, gyroscopes and magnetometers). Such systems have the advantages of having a lower cost and being more directly addressed towards measuring acceleration. Nowadays, they are present in most smartphones and motion-sensing game controllers. As we will see next, however, inertial measurement units by themselves still have certain limitations for our purposes.

Accelerometers measure acceleration through the inertial forces that are associated to it. Such forces are, for instance, those that press the passengers of a car backwards against their seat backs when the car accelerates forward. Now, these forces are of the same kind as gravity, which presses the same passengers down to their seats. The total effect is given by the difference vector

$$\mathbf{b} = \mathbf{a} - \mathbf{g}, \tag{8}$$

where $\mathbf{a}$ is the acceleration of the motion and $\mathbf{g}$ is the acceleration of gravity. We will refer to this vector $\mathbf{b}$ as the **sensed acceleration.** An accelerometer measures this vector $\mathbf{b}$, or equivalently, its opposite, sometimes called 'dynamic weight'. In order to emphasize the difference between $\mathbf{a}$ and $\mathbf{b}$, in the sequel we will refer to $\mathbf{a}$ as **kinematic acceleration.**

We are interested in the magnitude of the latter, i. e. the magnitude of $\mathbf{a} = \mathbf{b} + \mathbf{g}$. However, the computation is more involved than it seems. In fact, the accelerometer measures the sensed acceleration vector $\mathbf{b}$ in a coordinate frame that moves with the device, whereas the gravity acceleration $\mathbf{g}$ is known to us by its coordinates in a "fixed" frame. So, in order to work out the kinematic acceleration vector $\mathbf{a} = \mathbf{b} + \mathbf{g}$ and its magnitude we need to be able to convert coordinates from the moving frame to the fixed one, or vice versa. In other words, we need to know the orientation of the moving frame with respect to the fixed one. And we need that for every moment in a time sequence. This is why inertial measurement units contain a gyroscope. The latter measures the magnitude and direction of the instantaneous rate of rotation of the moving frame. Once again, the output consists of the coordinates of a vector in that moving frame. In principle, these data can be integrated so as to compute the instantatenous varying orientation of the moving frame with respect to a fixed one.

In practice, however, orientation errors accumulate over time (see for instance [9]), which makes it difficult to use such a procedure for measuring movement accents in dance from the data that are produced by the current strap-down inertial measurement units. A suitable benchmark in this connection is a slowly damped pendulum, where one should be able to detect and distinguish the different kinds of movement accents that we discussed in the preceding section. A system that is not able to match up the human perception of these accents (empirical observations 3–5) will miss also many other movement accents relevant to dance.

Our pendulum experiments used an inertial measurement unit manufactured by MEMSense LLC, more specifically the model BT 02-0300 F050. This device includes a tri-axial accelerometer with a dynamic range of 2 g and a tri-axial gyroscope with a dynamic range of 300 deg/s. These sensors are sampled with a frequency of 150 Hz, and the readings are transmitted wirelessly using the Bluetooth protocol.

Figure 3 shows the magnitude of the sensed acceleration obtained in that experiment. The shown time interval corresponds to angular amplitudes from about 60° at the beginning to about 40° at the end. According to the computations of § 3.3, in this interval the kinematic acceleration should exhibit a transition from the dominance of the swing accents to that of the rebound ones.

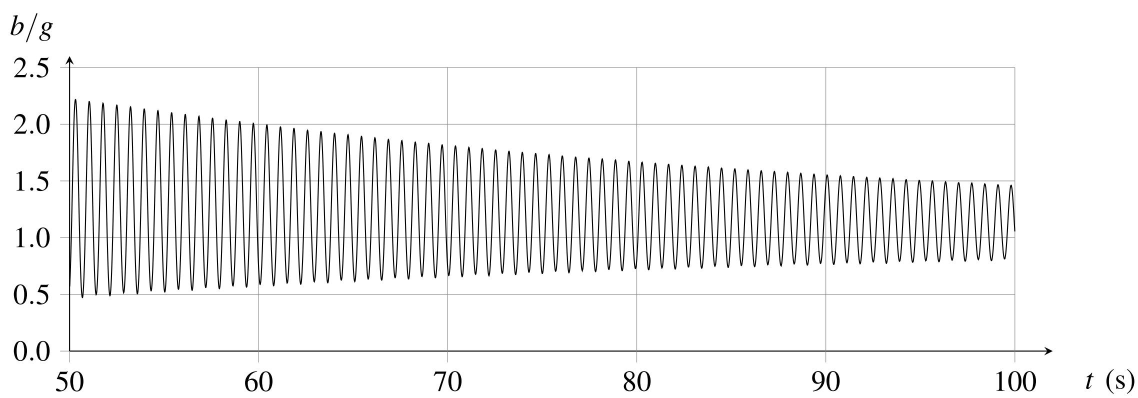

Figure 3. *Damped pendulum. Magnitude of the sensed acceleration.*

This transition shows up in figure 4, which has been computed by a pendulum-specific procedure that makes no use of the gyroscope data and thus avoids the errors of orientation drift. This procedure is based on the fact that the sensed acceleration vector $\mathbf{b}$ can be worked out to be colinear to the pendulum and to have the following signed magnitude in the centripetal direction:

$$b/g = 2(\eta - 1) + 3\cos\varphi. \tag{9}$$

This implies that the absolute maxima of b/g occur at $\varphi = 0$ and their value is $1 + 2\eta$. This information can be used to adjust a slowly decreasing function $\eta(t)$, thus modelling the progressive decay of the oscillations because of friction. Having adjusted this function, one can plug it into (9) and use this equation to derive $\cos\varphi(t)$ from the sensed acceleration data. Finally, introducing this information in (6) allows to compute the magnitude a of the kinematic acceleration as a function of time.

The resulting graph (figure 4) clearly shows two series of local maxima of a. They correspond respectively to the swing accents and the rebound ones. The swing accents are stronger than the rebound ones on the left-hand side of the figure, where η is larger and the oscillations have a larger amplitude. On the right-hand side we have the contrary situation. Notice that nothing of this kind was seen in the graph of the sensed acceleration magnitude (figure 3).

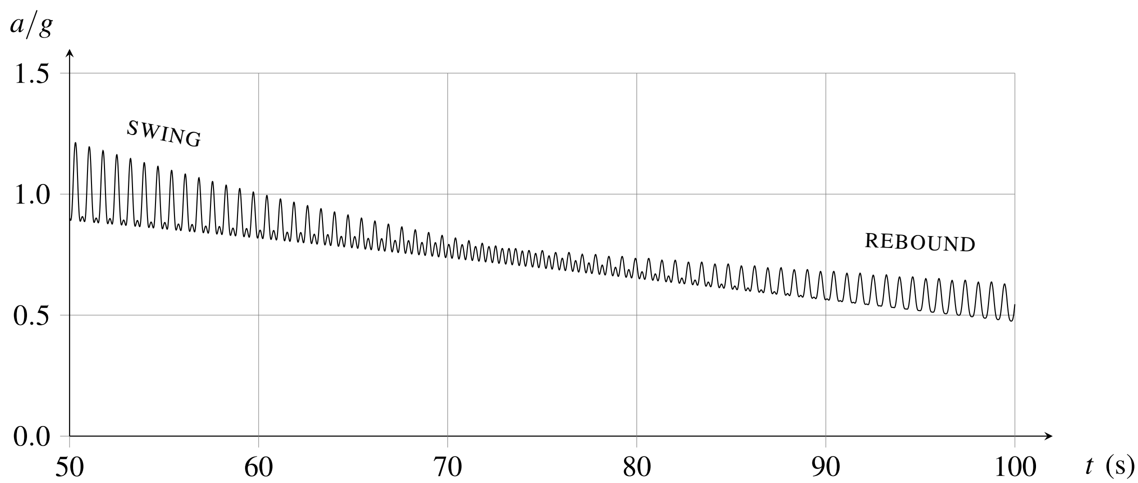

Figure 4. *Damped pendulum. Magnitude of the kinematic acceleration.*

Unfortunately, this picture does not show up easily in our computations of the kinematic acceleration through the gyroscope data. Nor does it appear either in the output of certain smartphone applications that claim to produce the kinematic acceleration. The reason of it seems to be a large orientation drift because of the highly dynamic character of the motion and the low accuracy of the strap-down gyroscopes.

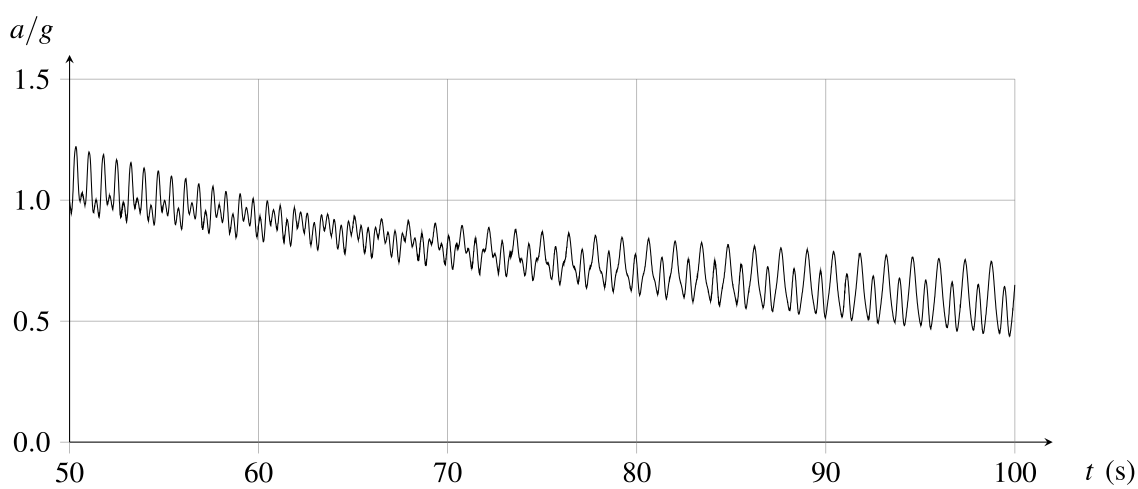

Figure 5. *Damped pendulum. Magnitude of the kinematic acceleration derived by means of gyroscope data.*

Having said that, figure 5 has been obtained by means of gyroscope data and it does exhibit a picture similar to that of figure 4. However, in order to produce it we kept the orientation drift to a minimum by starting the integration from a particular time instant near to 50 s where the orientation was known, namely a time instant where b presents a local maximum, which we know to correspond to a vertical position of the pendulum (so we still used some specific knowledge about the motion of the pendulum). As one can see, even in such advantageous circumstances the picture is much less clear than that of figure 4.

5 Conclusions

Rhythmic quality in dance is more objective than it seems. In this article we have restricted our attention to motions that consist in the translation of a single point. This simple case is appropriate for the movement of the body centre, which has a fundamental character in many dance genres.

We have pointed out certain empirical observations that help to analyse the human perception of movement accents in such motions. These empirical observations become explained if one postulates as accentuation variable the magnitude of the kinematic acceleration vector. One cannot say the same of other variables such as kinetic energy, its rate of change, nor the magnitude of the sensed acceleration vector.

Unfortunately, the current strap-down inertial measurement units are not accurate enough for correctly extracting the kinematic acceleration and identifying the movement accents of highly dynamic motions. While waiting for more accurate and better calibrated sensors, a more elaborate method would be required.

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