Zero-Hopf bifurcation in a modified Van der Pol-Duffing oscillator

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Abstract First we characterize the zero-Hopf bifurcation of a 3-dimensional modified Van der Pol-Duffing oscillator previously studied by several authors, providing sufficient conditions for the existence of three, two or one periodic solutions bifurcating from a zero-Hopf equilibrium, i.e. an equilibrium point whose eigenvalues are zero and a pair or purely imaginary. Second we determine the dynamics of this system near the infinity. In this way complete the study of the dynamics of this oscillator.

Keywords van der Pol-Duffing oscillator · zero-Hopf · dynamics at infinity

Mathematics Subject Classification (2010) MSC 37G15 · MSC 37G10 · MSC 34C07

1 Introduction

In [1,2] it was studied a modified Van der Pol-Duffing oscillator circuit that can be described by the following cubic polynomial system of differential equations

$$\begin{aligned} \dot{x} &= -m\left(x^3 - \mu x - y + \alpha\right), \\ \dot{y} &= x - y - z, \\ \dot{z} &= \beta y + \gamma z, \end{aligned} \tag{1}$$

where the state variables $(x, y, z) \in \mathbb{R}^3$ and the parameters $(m, \alpha, \mu, \gamma, \beta) \in \mathbb{R}^5$. As usual the dots denote differentiation with respect to the time t. Notice

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