PHASE PORTRAITS AND BIFURCATION DIAGRAMS FOR SOME HAMILTONIAN SYSTEMS WITH RATIONAL POTENTIALS

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ABSTRACT. By a suitable linear change of variables we provide the normal forms for the Hamiltonian systems

$$\dot{x} = H_y(x,y), \qquad \dot{y} = -H_x(x,y),$$

where the Hamiltonian function is a rational potential $H(x,y) = y^2/2 + P(y)/Q(x)$, where $P(y)$ and $Q(x)$ are polynomials of degree 1 or 2. Then we classify the global phase portraits of these Hamiltonian systems in the Poincaré disk and give their bifurcation diagrams. The main tools used in our proofs are the blow-up technique and the characterization of the separatrices.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

The global dynamics of the polynomial Hamiltonian systems is being studied currently, see for instance [3, 4, 5]. The authors of [2] studied the global phase portraits of all quadratic Hamiltonian systems. In [9] Gasull et al. studied the phase portraits of Hamiltonian systems of the form

$$(1) \qquad \dot{x} = H_y(x,y), \qquad \dot{y} = -H_x(x,y),$$

where $H(x,y) = (x^2 + y^2)/2 + H_{n+1}(x,y)$, and H_{n+1} is a homogeneous polynomial of degree $n + 1$. The authors of [6, 7] provided the bifurcation diagrams for the global phase portraits in the Poincaré disk of all Hamiltonian linear type centers and nilpotent centers of linear plus cubic homogeneous planar polynomial vector fields, respectively. Guillamon et al. [10] considered the separable Hamiltonian $H(x,y) = F(x) + G(y)$ and gave an algorithm to obtain their phase portraits. Martínez and Vidal [12] studied the dynamical behavior of the Hamiltonian systems, where the Hamiltonian function $H(x,y)$ has the particular form

$$H(x,y) = \frac{y^2}{2} + V(x),$$

where $V(x) = P(x)/Q(x)$ with $P(x)$ and $Q(x)$ are polynomials.

In this paper we consider the Hamiltonian vector fields (1) on $\mathbb{R}^2$, with Hamiltonian rational function

$$(2) \qquad H = H(x,y) = \frac{y^2}{2} + V(x,y) = \frac{y^2}{2} + \frac{P(y)}{Q(x)},$$