THE EXTENDED 16-TH HILBERT PROBLEM FOR PIECEWISE LINEAR SYSTEMS SEPARATED BY A NON-REGULAR LINE

JAUME LLIBRE[1] AND CLAUDIA VALLS[2]

ABSTRACT. We study the planar piecewise differential systems formed by two linear differential systems separated by a non-regular straight line such that both linear differential have no equilibria, neither real nor virtual. When the piecewise differential system is continuous, we show that these systems have no limit cycles. When the piecewise differential system is discontinuous, we provide upper bounds for the maximum number of limit cycles that these discontinuous piecewise differential systems can exhibit under the assumptions that at least one of the linear systems has divergence zero and when both systems have divergence zero we show that these upper bounds are reached. Hence we solve the extended 16th Hilbert problem for this class of piecewise differential systems.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

The study of piecewise linear differential systems goes back to Andronov, Vitt and Khaikin [1], and nowadays such systems still continue to receive the attention of many researchers, mainly due to their applications because they are widely used to model processes appearing in electronics, mechanics, economy, etc. (see for instance the books of di Bernardo et al. [7] and Simpson [37], the survey of Makarenkov and Lamb [35], as well as hundreds of references quoted in these last three works). One of the main interesting objects in the study of differential systems are limit cycles. A *limit cycle* is a periodic orbit of the differential system isolated in the set of all periodic orbits of the system. Limit cycles play an important role for explaining physical phenomena (see for instance the limit cycle of van der Pol equation [38, 39], or the one of the Belousov-Zhavotinskii model [3, 40], etc).

The *extended* 16*th Hilbert problem*, that is, to find an upper bound for the maximum number of limit cycles that a given class of differential systems can exhibit, is in general an unsolved problem. Only for very few classes of differential system this problem has been solved. For the class of continuous and discontinuous piecewise differential systems here studied, we can obtain the solution by using the first integrals of the two linear systems which form the discontinuous piecewise differential system separated by a non-regular line.

The simplest possible piecewise linear differential systems are the ones formed by two linear differential systems separated by a straight line. We note that for these apparently simple systems, when they are continuous, some serious work is necessary for proving that they have at most one limit cycle, see [10] and [27]. This solved the conjecture of Lum and Chua [34] done in 1990 that such continuous differential systems can have at most one limit cycle. The study of the maximum number of limit cycles of these systems, when they are discontinuous, still is an open question. Several authors have tried to determine the maximum number of limit cycles for this class of discontinuous piecewise differential systems. Up to now we know that there are discontinuous systems with at least three limit cycles, see, for instance, [2]–[5], [8], [11]–[19], [22]–[26], [28, 31]. But proving that

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