

## PERIODIC ORBITS FOR THE PERTURBED PLANAR CIRCULAR RESTRICTED 3-BODY PROBLEM

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**ABSTRACT.** We characterize when the classical first and second kind of periodic orbits of the planar circular restricted 3-body problem obtained by Poincaré, can be extended to perturbed planar circular restricted 3-body problems. We put special emphasis when the perturbed forces are due to zonal harmonic or to a solar sail.

**1. Introduction.** The circular restricted 3-body problem is considered the simplest non-integrable dynamical system in Celestial Mechanics. However it has many applications not only in the solar system but also in the studies of the stellar dynamics. Therefore many different dynamical astronomical systems can be studied within its frame. Thus, for instance this model is used in some space missions motion of spacecrafts in the Earth–Moon system, or between two planets. Also there are specific applications of the circular restricted 3-body problem in stellar systems for studying exoplanets around either one or both components of a binary star system, (see for instance [22]), and in particular for analyzing the existence of habitable zones in these exoplanets, see [21, 24, 26, 27, 30].

Many researchers pay their attention to present analytical studies on the circular restricted 3-body problem within the frame of some perturbed forces, due to oblateness, zonal harmonic coefficients, triaxial and radiation pressure effects. In

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