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A Computational Approach to the Fundamental Theorem of Asset Pricing in a Single-Period Market

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Abstract. We provide a new approach to the *Fundamental Theorem of Asset Pricing* based on the relation between the projection problem and equivalent least squares problem. More precisely, we use an iterative procedure in order to obtain solutions of a bounded least square problem. Under some conditions, this solution will give either the state price vector or the arbitrage opportunity of the problem under consideration.

Key words: asset pricing, arbitrage, mathematical finance

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1. Introduction

The basic idea of the whole pricing philosophy in finance consists of the construction of a linear functional π which strictly separates the arbitrage opportunities obtained by trading strategies. Moreover, this functional provides the state price vector necessary for pricing contingent claims in a financial market. In a more formal way, if we denote by M the linear subspace of all trading strategies and by $\mathbb{K}$ the nonnegative cone, then there are no arbitrage opportunities if and only if $M \cap \mathbb{K} = \{\mathbf{0}\}$. This result is known as the *Fundamental Theorem of Asset Pricing*. We remark that the existence of state price vectors follows by the *Separating Hyperplane Theorem* (Rockafellar, 1990) – a version of the Hahn–Banach theorem (see Duffie, 1992). However, the Hahn–Banach theorem is not a useful tool to construct the state price vector. Despite its practical importance, a computational approach to the calibration of the state price vector has received little attention in the mathematical finance theory in discrete time (see Sheldon (1996), Theorem 1.2, Bingham and Kiesel (1998), Theorem 1.4.1, and Pliska (1997), 1.16). We remark that the single period model is important because it provides much of the intuition that is necessary for more general models of financial markets and because it is possible to combine the result of the single period in order to prove the *Fundamental*