Nonlinear Analysis 75 (2012) 4114–4125

Contents lists available at SciVerse ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

On the global flow of a 3-dimensional Lotka–Volterra system

Justino Alavez-Ramírez [a], Gamaliel Blé [a], Víctor Castellanos [a], Jaume Llibre [b,*]

[a] División Académica de Ciencias Básicas, UJAT, Km 1 Carretera Cunduacán–Jalpa de Méndez, c.p. 86690 Cunduacán Tabasco, Mexico

[b] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

ARTICLE INFO

Article history:
Received 6 September 2011
Accepted 4 March 2012
Communicated by S. Ahmad

MSC:
37G15
37D45

Keywords:
α-limit
ω-limit
Phase portrait
Lotka–Volterra system
Black hole
Higgs field

ABSTRACT

In the study of the black holes with a Higgs field appears in a natural way the Lotka–Volterra differential system

$$\dot{x} = x(y-1), \qquad \dot{y} = y(1+y-2x^2-z^2), \qquad \dot{z} = zy,$$

in $\mathbb{R}^3$. Here we provide the qualitative analysis of the flow of this system describing the α-limit set and the ω-limit set of all orbits of this system in the whole Poincaré ball, i.e. we identify $\mathbb{R}^3$ with the interior of the unit ball of $\mathbb{R}^3$ centered at the origin and we extend analytically this flow to its boundary, i.e. to the infinity.

1. Introduction and statement of the main results

Breitenlohner et al. in their study of the black holes with a Higgs field reduced the relevant terms to the following Lotka–Volterra polynomial differential system in $\mathbb{R}^3$:

$$\dot{x} = x(y-1), \qquad \dot{y} = y(1+y-2x^2-z^2), \qquad \dot{z} = zy, \tag{1}$$

see for more details page 441 of [1]. These authors, analyzing the local motion around the z-axis which is formed by singular points, obtained information about the growing of the mass of a black hole with a Higgs field. We shall provide in this paper the complete description of the global dynamics of this differential system not only in $\mathbb{R}^3$, also in its compactification, controlling in this way the orbits which come or go to infinity. So this complete information about the dynamics of the differential system (1) must help to a better understanding of the the black holes with a Higgs field.

We want to mention that the global description of the flow of a differential system in $\mathbb{R}^3$ is usually very difficult and generally impossible. Here, using the compactification of Poincaré and the existence of an invariant of the system (roughly speaking, a first integral depending also on the time) such a description is possible for this differential system coming from the cosmology. The method followed for providing the global phase portrait of this system can be applied to many other systems, especially if they exhibit an invariant or a first integral.

The Lotka–Volterra systems are the differential systems of the form

$$\dot{x}_k = x_k f_k(x_1, \ldots, x_n), \quad \text{for } k = 1, \ldots, n.$$

* Corresponding author. Tel.: +34 935811303; fax: +34 935812790.
E-mail addresses: justino.alavez@ujat.mx (J. Alavez-Ramírez), gble@ujat.mx (G. Blé), vicas@ujat.mx (V. Castellanos), jllibre@mat.uab.cat (J. Llibre).

0362-546X/$ – see front matter
doi:10.1016/j.na.2012.03.002