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ON THE MAXIMUM NUMBER OF LIMIT CYCLES OF A CLASS OF GENERALIZED LIÉNARD DIFFERENTIAL SYSTEMS

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Applying the averaging theory of first, second and third order to one class generalized polynomial Liénard differential equations, we improve the known lower bounds for the maximum number of limit cycles that this class can exhibit.

Keywords: Liénard differential systems; polynomials; limit cycles.

1. Introduction and Statement of the Main Results

The generalized polynomial Liénard differential equation

$$\ddot{x} + f(x)\dot{x} + g(x) = 0,$$

or equivalently,

$$\dot{x} = y, \quad \dot{y} = -f(x)y - g(x), \tag{1}$$

was introduced in [Liénard, 1928]. Here the dot denotes the derivative with respect to the independent variable t, and $f(x)$ and $g(x)$ are polynomials in the variable x of degrees n and m, respectively. For this subclass of polynomial vector fields we have a simplified version of the 16th Hilbert's problem, see [Lins *et al.*, 1977] and [Smale, 1998].

Lins *et al.* [1977] studied the classical polynomial Liénard differential equations (1) with $g(x) = x$ and stated the following conjecture: *if $f(x)$ has degree $n \geq 1$ and $g(x) = x$, then (1) has at most $[n/2]$ limit cycles.* Here $[x]$ denotes the integer part function of $x \in \mathbb{R}$. They also proved the conjecture for $n = 1, 2$, and additionally they showed that there are systems (1) having at least $[n/2]$ limit cycles. For $n \geq 5$ this conjecture is not true as it has been proved recently by [Dumortier *et al.*, 2007] and by [De Maesschalck & Dumortier, 2011]. More recently the conjecture has been proved for $n = 3$ see [Li & Llibre, 2011]. In short, the conjecture only remains open for $n = 4$.

We note that a classical polynomial Liénard differential equation has a unique singular point. However, it is possible for generalized polynomial