SPATIAL CENTRAL CONFIGURATIONS FOR THE $1+4$ BODY PROBLEM

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Abstract

We deal with the central configurations of the $1+4$ body problem in space, i.e. we study the configurations without collision that are limit of central configurations of the 5 body problem in space, when the mass of one of the bodies goes to infinity. In every such limit the "small" masses are on a sphere whose center is the "big" mass. We suppose moreover that the four small masses are equal.

We prove that all these central configurations have at least one plane of symmetry. We first find four "very symmetric" central configurations: the configuration where the four small masses form a regular tetrahedron, two configurations with a three-fold symmetry (we mean invariant by a rotation of angle $2\pi/3$ around an axis), and a pyramidal configuration having for base a square formed by the small masses. We then obtain more information about the possible other central configurations. We prove that they are all of the "third kind", i.e. such that the four small masses are in the same hemisphere, and one of the small masses is located inside the spherical triangle formed by the other three.