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ORIGINAL ARTICLE

Periodic orbits for a class of galactic potentials

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Abstract In this work, applying general results from averaging theory, we find periodic orbits for a class of Hamiltonian systems H whose potential models the motion of elliptic galaxies.

Keywords Periodic orbit · Averaging theory · Hamiltonian system

1 Introduction and statement of the main results

Galactic dynamics is a branch of Astrophysics whose development started only around sixty years ago, when it was possible to have a view of the physical world beyond the integrable and near integrable systems (Contopoulos 2002). Even the importance of the analysis of galactic potentials, the global dynamics of galaxies is not a simple question and represents a big challenge for the researches in the field (Contopoulos 1988). Most of the work in the analysis of galaxies is numerical, in this paper we present an analytical technique, the averaging theory, which allows to find periodic orbits of a differential system.

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In the last years, great quantity of the research on galactic dynamics has been focused on models of elliptical galaxies. In most of these models the terms in the potential are of even order, so we have adopted this fact in the Hamiltonian system that we are analyzing. Another important point that appears in these kind of potentials is that the existence of periodic orbits is a useful tool for constructing new and more complicated self consistent models. One way to identify periodic orbits is to localize the central fixed points on the surfaces of constant energy. In Patsis and Zachilas (1990), the authors study the localization of periodic orbits and their linear stability for a particular two-component galactic potential. In fact, in our days the study of individual orbits in some galactic potentials is a new branch of galactic dynamics (see for instance the articles Caranicolas (2000), El-Sabaa and Sherief (1990), Greiner (1987)).

The calculation of particular orbits in some analytical potentials modeling elliptical galaxies, indicates that relatively small symmetry breaking corrections can increase dramatically the number of stochastic orbits, showing the importance of the study of perturbations of simple models (Habib et al. 1997). The class of potentials studied in this paper have not chosen with the aim of modeling some particular galaxies, our objective is to study systems which are generic in their basic properties.

In Pucacco et al. (2008), the authors study the galactic potential

$$H = \frac{1}{2}\left(P_X^2 + P_Y^2\right) + V\left(X^2, Y^2\right).$$

These kind of potentials are important in the modeling of elliptic galaxies, as for instance we can mention the potentials $V_L = \log\left(1 + X^2 + Y^2/q\right)$ and $V_C = \sqrt{1 + X^2 + Y^2/q} - 1$, where the parameter q gives the eccentricity of the elliptic