Minimal models for trees and graphs

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Abstract

The aim of the talk is to present a survey on the existence of minimal models for graph maps and its consequences.

Introduction

A classical problem in combinatorial dynamics is the following: Given a topological space X, a continuous map $f: X \longrightarrow X$ and a finite $f-$invariant set $A \subset X$, what can be said about the dynamics (periodic orbits, topological entropy, ...) of f in terms of $f\big|_A$? This question can be reworded as follows: What can be said about the dynamics of any continuous map $g: Y \longrightarrow Y$ for which there exists a homeomorphism $\varphi: X \longrightarrow Y$ such that $g \circ \varphi\big|_A = \varphi \circ f\big|_A$?

A classical (and well known) case is when X is a closed interval I of the real line. Indeed, if $f: I \longrightarrow I$ is a continuous map then intrinsic information can be obtained by considering the *"pattern"* of A which is characterized essentially by the permutation π_A induced by $f|_A$ (see [2] for a precise definition). To each pattern π_A we may associate a (non-unique) interval map f_π which admits a finite invariant set B, such that the permutation induced by $f_\pi|_B$ is π_A and f_π is monotone between consecutive points of B. Such a map is called a *canonical representative of* π_A, or a "connect–the–dots" map. It has the following important properties:

(A) f_π minimizes topological entropy within the class of interval maps admitting a periodic orbit whose pattern is π_A.

(B) f_π admits a Markov partition which gives a good "coding" to describe the dynamics of the map f_π. The topological entropy of f_π may be calculated from this partition.

(C) f_π is essentially unique.

(D) the pattern π_A forces a pattern ρ if and only if f_π has a periodic orbit whose pattern is ρ. We recall the definition that a pattern π_A *forces* a pattern π_B if and only if each map exhibiting the pattern π_A also exhibits the pattern π_B (see [2] and [15]). In this sense, the dynamics of f_π are minimal within the class of maps admitting a periodic orbit whose pattern is π_A.

One may make analogies with the study of the surface homeomorphisms where the canonical representatives are given by the Nielsen-Thurston Theorem [11, 17]. In this context the "pattern" or *braid type* $\mathrm{bt}(f, A)$ of a periodic orbit A of a surface homeomorphism $f: M \longrightarrow M$ is characterized by the isotopy class (up to conjugacy) of $f|_{M\setminus A}$ [8, 9, 14]. The permutation group arising in the interval case is now replaced