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ENTROPY OF TRANSITIVE TREE MAPS

LLUÍS ALSEDÀ, STEWART BALDWIN, JAUME LLIBRE and MICHAŁ MISIUREWICZ

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We obtain lower bounds for the topological entropy of transitive self-maps of trees, depending on the number of endpoints and on the number of edges of the tree. Copyright © 1996 Elsevier Science Ltd

1. INTRODUCTION

One of the central questions in the theory of dynamical systems is how to recognize chaos and how to see how large it is. One of the best methods of measuring chaoticity is by means of topological entropy of the system (see e.g. [1] for a definition). Then we can restate our question as: How can we get estimates for topological entropy from other properties of a system? The simplest kind of a dynamical system is formed by taking iterates of one map of a compact space into itself. It turns out that if this space is a tree then already such a weak property as transitivity (existence of a dense orbit) implies positive entropy (see [2]), and therefore chaoticity. Immediately, a problem arises: Are there any natural lower bounds for the entropy in this case?

The problem of obtaining lower bounds for the topological entropy of a transitive map has been considered by several authors for some special cases of one-dimensional spaces. Namely, Blokh in [3] proved that if the tree under consideration is a closed interval of the real line then $h(f) \geqslant (\log 2)/2$. In [4] the lower bounds for the topological entropy of transitive circle and star maps were obtained. In [5] the same problem has been considered for a class of transitive tree maps that arises naturally in the study of the homeomorphisms of the disk. For such class of maps the bound $(\log 2)/n$ for the topological entropy has been obtained, where n is the number of ends of the tree. In [6] a similar problem for the tree maps obtained from pseudo-Anosov diffeomorphisms of a punctured disk has been considered. The bound of the topological entropy obtained in this case is $(\log(1 + \sqrt{2})/k)$, where k is the number of punctures.

The aim of this paper is to obtain lower bounds for the topological entropy of transitive tree maps. To be more precise we have to introduce some notation.

By an *interval* we mean the closed interval $[0, 1]$ and any space homeomorphic to it. A *tree* is a connected space that is a union of finite number of intervals, but does not contain a subset homeomorphic to a circle.

A map $f\colon T \to T$ is called *transitive* if for every non-empty open subsets $U, V \subset T$ there is $n \geqslant 1$ such that $f^n(U) \cap V \neq \emptyset$. This is equivalent to the existence of a point with a dense orbit (see for instance [1]).

The *topological entropy* of a continuous map f from a tree to itself will be denoted by $h(f)$ (see e.g. [1] for a definition).

Let T be a tree. We define a number $L(T)$ as the infimum of topological entropies of transitive maps from T to T. Our aim is to give some reasonable estimates for $L(T)$. Of