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Torus maps and Nielsen numbers

LL. ALSEDÀ, S. BALDWIN, J. LLIBRE, R. SWANSON, W. SZLENK

ABSTRACT. The main results in this paper concern the minimal sets of periods possible in a given homotopy class of torus maps. For maps on the 2–torus, we provide a complete description of these minimal sets. A number of results on higher dimensional tori are also proved; including criteria for every map in a given homotopy class to have all periods, or all but finitely many periods.

1. Introduction

In dynamical systems, it is often the case that topological information can be used to study qualitative properties of the system. This article deals with the problem of determining the set of periods (of the periodic orbits) of a torus map given the homotopy class of the map. To fix terminology, suppose f is a continuous self–map on the torus T^m. A *fixed point of* f is a point x in T^m such that $f(x) = x$. We will call x a *periodic point of period* n if x is a fixed point of f^n but is not fixed by any f^k, for $1 \leq k < n$.

Denote by $Per(f)$ the set of natural numbers corresponding to periods of periodic orbits of f.

Our aim is to provide a description of the minimal set of periods (see below) attained within the homotopy class of a given torus map $f : T^m \to T^m$. We also present a few results, described below, in a more general setting.

Toward this end, it is convenient to distinguish among several subsets of the natural numbers N. First, there is the set of periods, $Per(f)$, mentioned above. When the mapping $g : T^m \to T^m$ is homotopic to f, we shall write $g \simeq f$. Define the *minimal set of periods* of f to be the set

$$MPer(f) = \bigcap_{g \simeq f} Per(g).$$

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The complete version of this paper will be submitted elsewhere.