Papers

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ON THE STRUCTURE OF THE ω-LIMIT SETS FOR CONTINUOUS MAPS OF THE INTERVAL

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We introduce the notion of the center of a point for discrete dynamical systems and we study its properties for continuous interval maps. It is known that the Birkhoff center of any such map has depth at most 2. Contrary to this, we show that if a map has positive topological entropy then, for any countable ordinal α, there is a point $x_\alpha \in I$ such that its center has depth at least α. This improves a result by [Sharkovskii, 1966].

1. Introduction

The (*Birkhoff*) *center* of a discrete dynamical system is defined to be the closure of the set of its recurrent points and so it contains all the information about its "recurrent objects". In this paper we introduce the notion of the *center of a point* for discrete dynamical systems, and we study its properties for continuous interval maps with positive topological entropy. In [Simó, 1997] the same notion for a special kind of vector fields was studied and results in the spirit of ours were obtained. Earlier, a question posed to A. R. D. Mathias on the depth of the center of a point led to papers [Mathias, 1995, 1996, 1997] on the subject.

Throughout this paper, $\mathbb{N}$ and $\mathbb{Z}^+$ will denote the set of positive and non-negative integers, respectively. Let X be a compact metric space and let $f : X \longrightarrow X$ be a continuous map. The *ω-limit set of* x, denoted by $\omega_f(x)$, is the set of points $y \in X$ for which there exists a sequence of positive integers $\{n_k\}_{k\in\mathbb{N}}$ tending to infinity such that $\lim_{k\to\infty} f^{n_k}(x) = y$. For any ordinal α we define by transfinite induction

$$W_f^0(x) = w_f(x)\,,$$

$$W_f^{\alpha+1}(x) = \bigcup_{z\in w_f^\alpha(x)} \omega_f(z)\,,$$

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