Journal of Difference Equations and Applications
Vol. 14, No. 9, September 2008, 987–989

Taylor & Francis
Taylor & Francis Group

Open Problem

An open question on the Lyapunov exponents for two-dimensional quasiperiodically forced maps

Lluís Alsedà* and Sara Costa

Departament de Matemàtiques, Facultat de Ciències, Edifici C. Universitat Autònoma de Barcelona, Cerdanyola del Vallès, Spain

(*Received 6 May 2008; final version received 7 May 2008*)

Keywords: open problem; Lyapunov exponents; Oseledets theorem; lim sup; two-dimensional quasiperiodically forced skew products

AMS Subject Classification: 37E05; 37E99; 37B55; 37C70; 37H15

1. Introduction

Oseledets theorem implies that for a.e. point with respect to any invariant measure, the sum of the Lyapunov exponents is

$$\lim_{n\to\infty} \frac{1}{n} \log|\det(DF^n(x))|,$$

where $x_{n+1} = F(x_n)$ is the system under consideration. We are interested in the validity of this assertion for every point in the space, when F is a two-dimensional quasiperiodically forced skew product. In that case, 'lim' has to be replaced by 'lim sup' in the above expression and in the computation of the Lyapunov exponents.

2. Basic definitions

Let us consider a differentiable map $F{:}M \to M$ defined on a smooth compact Riemannian k-dimensional manifold (we do not require F to be invertible, since we will only consider forward orbits). For each $x \in M$ and $v \in T_xM$ the Lyapunov exponent at x in the direction v is defined by:

$$\lambda(x, v) = \limsup_{n\to\infty} \frac{1}{n} \log\|DF^n(x)\cdot v\| \in [-\infty, \infty), \tag{1}$$

where $DF^n(x)$ denotes the differential of nth iterate of F evaluated at x and $\|\cdot\|$ is a norm on T_xM. Since, T_xM is a k-dimensional vectorial space, all its norms are equivalent, and thus $\lambda(x,v)$ does not depend on the chosen norm.

It can be proved that for each $x \in M$, $\lambda(x,\cdot)$ takes finitely many different values: $-\infty = \lambda_0^x < \lambda_1^x < \cdots < \lambda_{s_x}^x$. In fact, there exists a linear filtration of T_xM, $\mathcal{V} = \{V_i{:}i = 0,$

* Corresponding author. Email: alseda@mat.uab.cat

ISSN 1023-6198 print/ISSN 1563-5120 online

DOI: 10.1080/10236190802186860
http://www.informaworld.com