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On the definition of strange nonchaotic attractor

by

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Dedicated to Michał Misiurewicz on the occasion of his 60th birthday

Abstract. The aim of this paper is twofold. On the one hand, we want to discuss some methodological issues related to the notion of strange nonchaotic attractor. On the other hand, we want to formulate a precise definition of this kind of attractor, which is "observable" in the physical sense and, in the two-dimensional setting, includes the well known models proposed by Grebogi et al. and by Keller, and a wide range of other examples proposed in the literature. Furthermore, we analytically prove that a whole family of two-dimensional quasiperiodic skew products defined on $\mathbb{S}^1 \times \mathbb{R}$ have strange nonchaotic attractors. As a corollary we show analytically that the system proposed by Grebogi et al. has a strange nonchaotic attractor.

1. Introduction. The notion of *strange nonchaotic attractor* (briefly SNA) was introduced ([1]) by Grebogi et al. in [10] when studying attractors of quasiperiodically forced skew products of the form

$$\begin{cases} \theta_{n+1} = \theta_n + \omega \ (\mathrm{mod}\ 1), \\ x_{n+1} = \psi(\theta_n, x_n), \end{cases} \tag{1}$$

where $x \in \mathbb{R}$, $\theta \in \mathbb{S}^1$ and $\omega \in \mathbb{R} \setminus \mathbb{Q}$. One of the two examples they considered consisted in taking $\psi(\theta, x) = 2\sigma \cos(2\pi\theta) \tanh(x)$ in the above system (see Figure 1 for a picture of the attractor of an instance of this system):

$$\begin{cases} \theta_{n+1} = \theta_n + \omega \ (\mathrm{mod}\ 1), \\ x_{n+1} = 2\sigma \cos(2\pi\theta_n) \tanh(x_n). \end{cases} \tag{2}$$

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([1]) This kind of attractor had already been studied in the literature much earlier than the term SNA was coined. For example in [18, 19, 24] constructions of flows containing strange nonchaotic attractors can be found. Also, in the last part of this paper, we see that some well known one-dimensional attractors are SNA's.

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