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Dynamics on Hubbard trees

by

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Abstract. It is well known that the Hubbard tree of a postcritically finite complex polynomial contains all the combinatorial information on the polynomial. In fact, an abstract Hubbard tree as defined in [23] uniquely determines the polynomial up to affine conjugation. In this paper we give necessary and sufficient conditions enabling one to deduce directly from the restriction of a quadratic Misiurewicz polynomial to its Hubbard tree whether the polynomial is renormalizable, and in this case, of which type. Moreover, we study dynamical features such as entropy, transitivity or periodic structure of the polynomial restricted to the Hubbard tree, and compare them with the properties of the polynomial on its Julia set. In other words, we want to study how much of the "dynamical information" about the polynomial is captured by the Hubbard tree.

1. Introduction. In this paper we deal with Hubbard trees of Misiurewicz polynomials, for the most part of degree two, and the dynamical properties of such a polynomial f when restricted to its Hubbard tree $H = H(f)$. A Hubbard tree and the restricted map catch the essence of the dynamics of f. Indeed, from the combinatorial information given by a map on an abstract Hubbard tree (satisfying certain conditions) one can obtain the affine class of the actual polynomial realizing the tree as its Hubbard tree. But since it is easier to deal with the dynamics on the tree, it is of interest to describe how one reads off properties of the polynomial f directly from the dynamics of the tree map $f|_{H(f)}$.

The main results of the paper (see Subsection 1.5) give necessary and sufficient conditions enabling one to deduce directly from $f|_{H(f)}$ whether the polynomial is renormalizable, and of which type. Renormalization is a very important concept in holomorphic dynamics; it is therefore of interest to have a purely combinatorial characterization of this notion. Our results also show that other dynamical properties of the polynomial on its Julia set, such as density of periodic points, total transitivity or maximal topological

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