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DEVIL'S STAIRCAISE ROUTE TO CHAOS IN A FORCED RELAXATION OSCILLATOR

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1. Introduction and statement of the main result.

In this paper we describe the behaviour of certain sets of solutions of an oscillator of the Van der Pol type with sinusoidal forcing term. The original problem was proposed by Van der Pol [16] in the study of an electrical circuit with a triode valve. Later on, Van der Pol and Van der Mark [17] studied the forced relaxation oscillator in a circuit as the one in figure 1.1. They analyzed the frequency of the circuit as a function of the capacitance C. While increasing C from its initial value they observed that the electrical system takes a period being a multiple of the forcing period and that, for certain parameter values, two different subharmonics may coexist. Furthermore, there are regions where no subharmonics are detected. Plotting the frequency of the circuit against the capacitance they obtained a staircase structure as shown in figure 1.1.

Recently, Kennedy, Krieg and Chua [10] working with a modern version of the Van der Pol and Van der Mark's circuit observed the appearance of secondary staircases. These staircases present a well-known geometric structure called « the Devil's Staircase » (which, roughly speaking, can be defined as the graph of a non-decreasing continuous map with the property that the preimage of any rational number is a closed interval and the preimage of any irrational number is a point). These secondary staircases give the route from the non-chaotic behaviour to the chaotic one in the electrical circuit.

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