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ON THE TOPOLOGICAL DYNAMICS AND PHASE-LOCKING RENORMALIZATION OF LORENZ-LIKE MAPS

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1. Introduction.

An important question in the theory of dynamical systems is whether small scale geometric properties of a dynamical system are determined by the combinatorial properties of the system. In the case of unimodal maps such universality was discovered by Coullet and Tresser and Feigenbaum independently. In both cases renormalization ideas that arose from statistical physics were used. The idea is to study dynamical or parameter scaling laws by iterating certain renormalization operator acting in the space of dynamical systems. This operator acts as a microscope: the image under renormalization is another map in the class of maps under consideration which describes the geometry and dynamics on a smaller scale. In the case of unimodal maps the universality was understood by conjecturing that the renormalization operator has a unique hyperbolic fixed point whose invariant manifolds have certain "good" properties.

On the other hand, in the recent years some attention has been paid to Lorenz maps and β-transformations due to the fact that they help in understanding the dynamics of important three dimensional flows (see [22] and [18]). Lorenz maps were obtained by Lorenz when studying geometric models of the Lorenz equations (see [13], [10], [11], [21] and [22]). A *Lorenz*

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