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ON THE STRUCTURE OF THE KNEADING SPACE OF BIMODAL DEGREE ONE CIRCLE MAPS

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Dedicated to the memory of Valery S. Melnik

In this paper, we introduce an index space and two $\star$-like operators that can be used to describe bifurcations for parametrized families of degree one circle maps. Using these topological tools, we give a description of the kneading space, that is, the set of all dynamical combinatorial types for the class of all bimodal degree one circle maps considered as dynamical systems.

Keywords: Kneading theory; degree one circle maps; bifurcation space.

1. Introduction and Statement of Main Theorem

For continuous maps on the interval with finitely many monotonicity intervals, the kneading theory developed by Milnor and Thurston [1988] gives a symbolic description of the dynamics of these maps. This description is given in terms of the kneading invariants which essentially consist of the symbolic orbits of the turning points of the map under consideration. Moreover, this theory also gives a classification of all such maps through these invariants. For continuous bimodal degree one circle maps similar invariants were introduced by Alsedà and Mañosas [1990]. In that paper, the first part of the program just described was carried through, and relations between the circle maps invariants and the rotation interval were elucidated. Later on, in [Alsedà & Falcó, 1997, Theorem A] the set of all these kneading invariants (the *kneading space*) was characterized. The main goal of this paper is to give a description of the kneading space of the bimodal degree one circle maps using some self-similarity operators which allow us to identify certain subsets with known structure. To state this description we need the appropriate notation. This paper is, in some sense, a continuation of [Alsedà & Falcó, 1997] and we use heavily the notation and results from that paper. Although we have tried to make this paper self-contained in the introduction we have repeated certain definitions in [Alsedà & Falcó, 1997] for readability.

As it is usual, instead of working with the circle maps themselves we will rather use their liftings to the universal covering space $\mathbb{R}$. To this end, we