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SIMPLE AND COMPLEX DYNAMICS FOR CIRCLE MAPS*

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Abstract

The continuous self maps of a closed interval of the real line with zero topological entropy can be characterized in terms of the dynamics of the map on its chain recurrent set. In this paper we extend this characterization to continuous self maps of the circle. We show that, for these maps, the chain recurrent set can exhibit a new dynamic behaviour which is specific of the circle maps of degree one.

1. Introduction.

The aim of this paper is to extend the characterization of the complex and simple interval maps (in the sense of positive or zero topological entropy respectively) to circle maps. We shall start by stating this characterization for interval maps for completeness (see Theorem A). To do it we have to introduce the appropriate notation.

Let f be a map from a topological space X into itself. We shall denote by f^n the map $f \circ f \circ \ldots \circ f$ n times (if $n = 0$ we set $f^n = \mathrm{Id}$).

Let now f be an interval map (that is, a continuous map from a closed interval I of the real line into itself). We say that f has a *horseshoe* if there exist $n > 0$ and two closed intervals $I_1, I_2 \subset I$ with pairwise disjoint interiors such that $I_1 \cup I_2 \subset f^n(I_1)$ and $I_1 \cup I_2 \subset f^n(I_2)$.

The above condition was used for the first time by Sharkovskiĭ(see [**13**]) and has been used widely in the study of interval maps (see [**9**], [4] and [**12**]). The name of horseshoe was given to this condition by Misiurewicz

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