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"CONNECT-THE-DOTS" TREE MAPS

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We announce the main results of work in progress on piecewise monotone models for patterns of tree maps. More precisely, we define a notion of pattern for tree maps, and given such a pattern, we construct a tree and a piecewise monotone map on this tree with the same pattern. This piecewise monotone model has the least entropy among all models exhibiting the given pattern and has "minimal dynamics". We also give a formula to compute this minimal entropy directly from the pattern. These results generalize the known results for interval maps and the results from Li & Ye [1993].

The aim of this paper is to construct tree maps having minimal dynamics amongst all tree maps admitting a given behavior. By a tree map we understand any continuous map from a finite tree to itself (later on we give a precise definition of a tree). To motivate our work we begin by recalling the situation in the case of interval maps. In this case the "pattern" of a finite invariant subset P is essentially the permutation induced by the map on P. There is a simple candidate for a piecewise monotone map f such that $f|_P$ coincides with the given map restricted to P, namely the so-called "connect-the-dots" map. It can be thought of as the map such that for each interval $[a, b]$ where a, b are consecutive points of P we have that $f|_{[a,b]}$ is a homeomorphism onto its image. Such a map has the following properties:

(a) f has the least entropy among all interval maps having a periodic orbit with the same pattern as P. That is, the "connect-the-dots" models are the simplest with respect to horseshoes (recall that the entropy of an interval map is due to horseshoes; see Misiurewicz & Szlenk [1980], Misiurewicz [1980] and Llibre & Misiurewicz [1993]).