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PATTERNS OF TREE MAPS [1]

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Abstract. We define a notion of *pattern* for finite invariant sets of continuous maps of finite trees. A pattern is essentially a homotopy class relative to the finite invariant set. Given such a pattern, we prove that the class of tree maps which exhibit this pattern admits a canonical representative, that is a tree and a continuous map on this tree, which satisfies several minimality properties. For instance, it minimizes topological entropy in its class and its dynamics are minimal in a sense to be defined. We also give a formula to compute the minimal topological entropy directly from the combinatorial data of the pattern. These results generalize the known results for interval maps and the results from [6].

1. Introduction

One-dimensional dynamics have been studied intensively during the last three decades. The characterization by Sharkovskii [8] of the possible sets of periods of continuous maps of an interval I has generated many questions and results (for instance see [2] for a review). If $f : I \longrightarrow I$ is such a map and if A is a finite invariant set of f then intrinsic information can be obtained by considering the *"pattern"* of A which is characterized essentially by the permutation π_A induced by $f|_A$ (see [3] and [7] for a precise definition). To each pattern π_A we may associate a (non-unique) interval map f_π which admits a finite invariant set B, such that the permutation induced by $f_\pi|_B$ is π_A and f_π is monotone between consecutive points of B.