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All solenoids of piecewise smooth maps are period doubling

by

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Abstract. We show that piecewise smooth maps with a finite number of pieces of monotonicity and nowhere vanishing Lipschitz continuous derivative can have only period doubling solenoids. The proof is based on the fact that if $p_1 < \ldots < p_n$ is a periodic orbit of a continuous map f then there is a union set $\{q_1, \ldots, q_{n-1}\}$ of some periodic orbits of f such that $p_i < q_i < p_{i+1}$ for any i.

1. Introduction. Solenoids are important in one-dimensional discrete dynamical systems because they play a key role in the description of the asymptotic behaviour of a large class of maps from the family $C(I)$ of continuous maps from a real compact interval I into itself. Namely, if $f \in C(I)$ is piecewise strictly monotone and smooth enough then there is a residual set R such that for any $x \in R$ the set of limit points of the sequence $(f^n(x))_{n=0}^{\infty}$ is either a periodic orbit, a finite union of closed intervals or a solenoid. Regarding this, the reader may for example wish to check [12], [10], to which we also refer for the basic background and terminology. (To ease a first reading of the Introduction, all definitions related to solenoids and the description of their relevant properties are postponed until the beginning

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