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MAXIMIZING ENTROPY OF CYCLES ON TREES

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Abstract. In this paper we give a partial characterization of the periodic tree patterns of maximum entropy for a given period. More precisely, we prove that each periodic pattern with maximal entropy is irreducible (has no block structures) and simplicial (any vertex belongs to the periodic orbit). Moreover, we also prove that it is maximodal in the sense that every point of the periodic orbit is a "turning point".

1. **Introduction.**

A *pattern* is a classical and well studied object in the theory of one-dimensional combinatorial dynamics. Given a topological space X and a continuous map $f\colon X \longrightarrow X$ which is known to exhibit a finite invariant set P, the *pattern* of P is a combinatorial object that encodes information about both the relative positions of the points of P inside the space X and the way these positions are permuted under the action of $f\big|_P$.

When X is an interval, the pattern of P can be identified with a permutation π in a natural way: set $P = \{p_1, p_2, \ldots, p_n\}$ with $p_1 < p_2 < \ldots < p_n$ and define $\pi\colon \{1, 2, \ldots, n\} \longrightarrow \{1, 2, \ldots, n\}$ as $\pi(i) = j$ if and only if $f(p_i) = p_j$. If P is a periodic orbit then π is a cyclic permutation and the pattern is called *cyclic* or *periodic*. The notion of pattern for maps of the interval has its roots in the well known Sharkovskii's Theorem [21, 23], but it was formalized and developed by Misiurewicz and Nitecki [19] in the early 1990s building on a previous work by Baldwin [9].

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