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A NOTE ON THE PERIODIC ORBITS AND TOPOLOGICAL ENTROPY OF GRAPH MAPS

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ABSTRACT. This paper deals with the relationship between the periodic orbits of continuous maps on graphs and the topological entropy of the map. We show that the topological entropy of a graph map can be approximated by the entropy of its periodic orbits.

1. INTRODUCTION

The notion of topological entropy appeared early in the sixties (see [1]). It is defined for continuous maps on compact metric spaces and is a quantitative measure of the dynamical complexity of the map. It is an important topological invariant.

There are some properties of the dynamical behavior of the maps which are controlled by the topological entropy. For instance, it measures the exponential growth rate, when n tends to infinity, of the number of different orbits of length n if we use certain precision to distinguish two orbits (see [6]). For a piecewise monotone map f of the interval, it measures also the exponential rate of increase with n of the number of maximal intervals of monotonicity of f^n (see [10]).

We are interested in relating periodic orbits and topological entropy. For continuous maps on the interval, to every periodic orbit P of f we can associate a number $h(P)$ which is the topological entropy of the "connect-the-dots" map corresponding to P or the "linearization" of P. In fact, this entropy corresponds to the infimum of the entropies of all maps exhibiting orbits with the same combinatorics as P (see Corollary 4.4.7 of [2]).

In the interval case it is possible to show that the entropy of any map f is the supremum of the values $h(P)$ corresponding to all the periodic orbits P of f. Furthermore, for each n, we can take this supremum only over the orbits of period $k > n$. This result was stated by Takahashi [12] and proved with the assumption that f is piecewise monotone. In the general case it was also proved in an independent way by Block and Coven [5] and Misiurewicz and Nitecki [9].

Since the topological entropy is usually considered as a measure of the degree of chaos, a natural problem is developing algorithms for calculating it (see [7], [11] or [4]). These algorithms are based on different properties of the entropy and some of

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