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ON THE MINIMAL MODELS FOR GRAPH MAPS

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The aim of this paper is to present a survey on the existence of minimal models for graph maps and its consequences.

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1. Introduction

A classical problem in combinatorial dynamics is the following: Given a topological space X, a continuous map $f : X \to X$ and a finite f-invariant set $A \subset X$, what can be said about the dynamics (periodic orbits, topological entropy, ...) of f in terms of $f|_A$? This question can be reworded as follows: What can be said about the dynamics of any continuous map $g : Y \to Y$ for which there exists a homeomorphism $\varphi : X \to Y$ such that $g \circ \varphi|_A = \varphi \circ f|_A$?

A classical (and well known) case is when X is a closed interval I of the real line. Indeed, if $f : I \to I$ is a continuous map then intrinsic information can be obtained by considering the "*pattern*" of A which is characterized essentially by the permutation π_A induced by $f|_A$ (see [Alsedà *et al.*, 2000a] for a precise definition). To each pattern π_A we may associate a (nonunique) interval map f_π which admits a finite invariant set B, such that the permutation induced by $f_\pi|_B$ is π_A and f_π is monotone between consecutive points of B. Such a map is called a *canonical representative of* π_A, or a "connect-the-dots" map. It has the following important properties:

(A) f_π minimizes topological entropy within the class of interval maps admitting a periodic orbit whose pattern is π_A.
(B) f_π admits a Markov partition which gives a good "coding" to describe the dynamics of the map f_π. The topological entropy of f_π may be calculated from this partition.
(C) f_π is essentially unique.
(D) the pattern π_A forces a pattern ρ if and only if f_π has a periodic orbit whose pattern is ρ. We

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