Papers

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SETS OF PERIODS FOR PIECEWISE MONOTONE TREE MAPS

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We study the set of periods of tree maps $f : T \to T$ which are monotone between any two consecutive points of a fixed periodic orbit P. This set is characterized in terms of some integers which depend only on the combinatorics of $f|_P$ and the topological structure of T. In particular, a *type* $p \geq 1$ of P is defined as a generalization of the notion introduced by Baldwin in his characterization of the set of periods of star maps. It follows that there exists a divisor k of the period of P such that if the set of periods of f is not finite then it contains either all the multiples of kp or an initial segment of the ${}_{kp}\geq$ Baldwin's ordering, except for a finite set which is explicitly bounded. Conversely, examples are given where f has precisely these sets of periods.

Keywords: Tree maps; periodic orbits; set of periods.

1. Introduction

In this paper we deal with the problem of determining which are the possible sizes of the periodic orbits that appear by iterating a continuous map defined on a tree. For some particular cases (interval and star), several well known results establish that if a continuous map exhibits a periodic orbit which verifies some combinatorial properties then we can determine a set which is a lower bound of the set of periods of the map.

The widely known Sharkovskii's Theorem (see [Sharkovskii, 1964]) studying the set of periods of any continuous map from an interval of the real line into itself was the first remarkable result in this setting. In order to state it, we introduce the Sharkovskii ordering $\trianglerighteq$ (the symbols $\trianglelefteq$, $\triangleleft$ and $\triangleright$ will be understood in the natural way) in the set

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